Using Variable-Fidelity Models in Multidisciplinary Design Optimization

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The Engineering Multidisciplinary Optimization (MDO) Problem...

...can be stated as

$$\begin{aligned} \textbf{minimize} \quad & f(x, u(x)) \\ \textbf{subject to} \quad & h(x, u(x)) = 0 \\ & g(x, u(x)) \geq 0, \end{aligned}$$

where, given design variables x, the state variables $u(x)$ are defined via

$$A(x, u(x)) = \begin{pmatrix} A_1(x, u_1(x), \ldots, u_N(x)) \\ \vdots \\ A_N(x, u_1(x), \ldots, u_N(x)) \end{pmatrix} = 0,$$

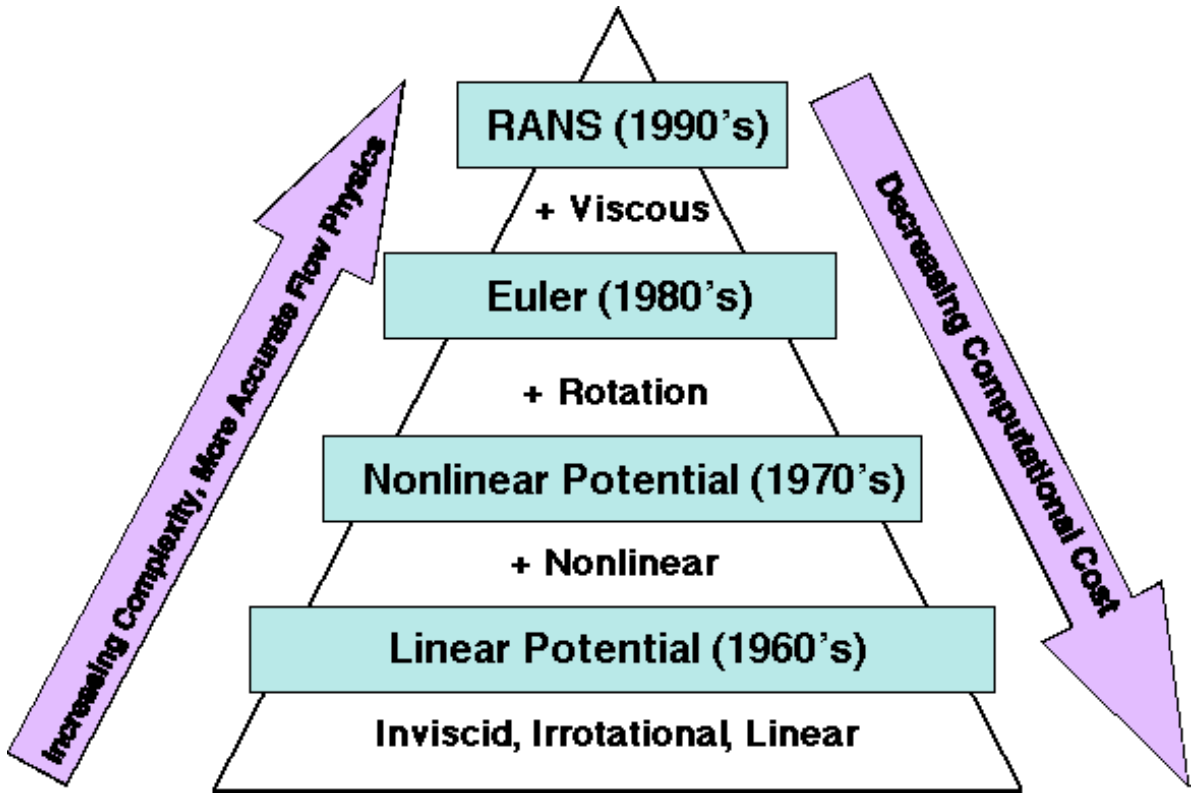
The blocks of the system usually represent the state equations for the disciplinary analyses and the necessary interdisciplinary couplings.

Motivation

- **Address computational expense of repeated use of high-fidelity models**
  - **Solutions of coupled PDE typically required at each iteration**
  - **For uncoupled problem formulations, the number of function evaluations typically rises**
  - **The difficulty is not likely to disappear with improvements in computational technology**
  - **Use of lower-fidelity models alone does not guarantee improvement in higher-fidelity design**
  - **Variable-fidelity models in use for a long time**
- **Allow for easier integration of disciplines in MDO**
- **Allow for interactive design**
- **Demonstrate feasibility of proposed methods on engineering problems**

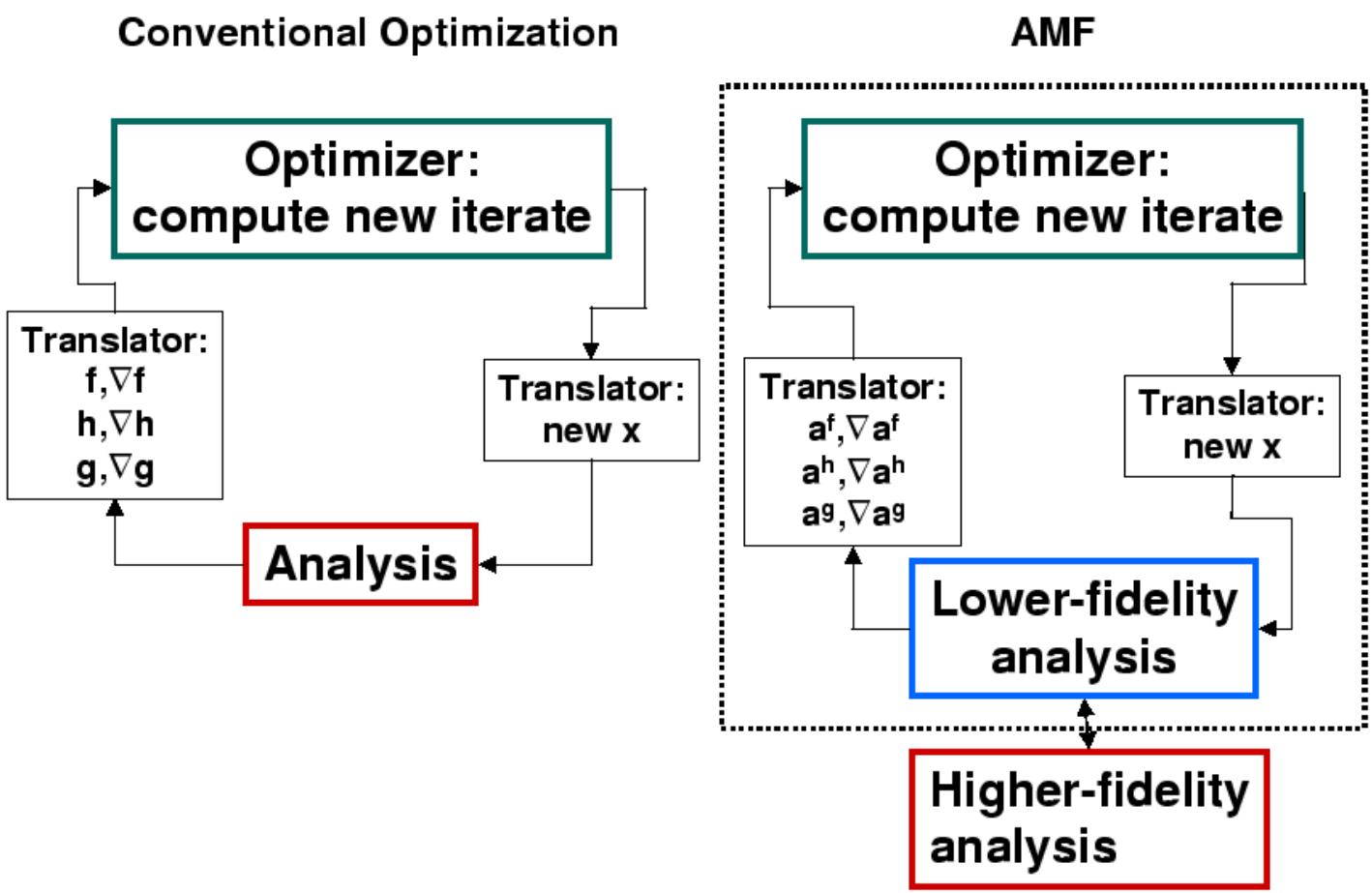
Example: Variable-Fidelity Computational Models in CFD

- **Variable-fidelity physical models (Jameson, 1997)**



- **A single physical model evaluated on meshes of varying refinement**
- **Polynomial approximations such as RSM, kriging**

Conventional Optimization vs. Approximation Management



Some Related Work

- **Overview of approximation concepts in structural design, Barthelemy and Haftka 1993**
- **Research conducted or supported at NASA Langley:**
  - **Design-oriented analysis, Gumbert et al., Silva et al. (NASA LaRC), Haftka at al. (University of Florida/VPI)**
  - ***A posteriori* error bounds for outputs of PDE and sensitivity derivatives of outputs, Lewis (ICASE), Patera et al. (MIT)**
  - **Managing models/approximations in optimization, Alexandrov et al. (NASA LaRC)**
- **Managing approximations in optimization, Booker et al. (Boeing/IBM/Rice/W&M)**

A First-Order AMF for Constrained Optimization

- **Potentially as many AMF's as there are optimization algorithms**
- **The underlying algorithm - MAESTRO (Alexandrov '93, Alexandrov and Dennis '98)**
- **Control "the amount" of optimization by varying the size of the trust region**
- **Not necessary to change physical models to obtain convergence**
- **When other models available, guidance on alternating**
- **Easily applicable to MDO problems**

AMF: model of constraints and substep

- **Consider minimize$\{f(x) : h(x) = 0\}$, where f and h are expensive**
- **Let x_c be the current iterate and Δ_c be the trust-region radius; set $z_0 = x_c$**
- **At x_c, select a model of the constraints a_c^h that satisfies:**

$$\begin{aligned} a_c^h(x_c) &= h(x_c) \\ \nabla a_c^h(x_c) &= \nabla h(x_c) \end{aligned}$$

- **Find a substep s_1 that approximately solves:**

$$\begin{aligned} &\textbf{minimize} && a_c^h(z_0 + s) \\ &\textbf{subject to} && \| s \| \leq \theta \Delta_c, \quad \theta \in (0.5, 0.6) \end{aligned}$$

- **Set $z_1 = z_0 + s_1$**

AMF: model of objective and substep

- **Select a model a_c^f of the objective function that satisfies:**

$$\begin{aligned} a_c^f(x_c + s_1) &= f(x_c + s_1) \\ \nabla a_c^f(x_c + s_1) &= \nabla f(x_c + s_1) \end{aligned}$$

- **Find s_2 that approximately solves:**

$$\begin{aligned} &\textbf{minimize} && a_c^f(z_1 + s) \\ &\textbf{subject to} && \| s \| \leq \sqrt{\Delta_c^2 - \|s_1\|^2} \end{aligned}$$

- **Set $s_c = s_1 + s_2$**

- **An extension of MAESTRO—the Gauss-Newton model of the constraints and the quadratic model of the objective replaced by general models that satisfy first-order consistency conditions**

AMF: evaluating the step / updating

- **Merit function:** $\mathcal{P}(x;\rho) \equiv f(x) + \rho \parallel h(x) \parallel^2$ **or** $\mathcal{L}(x,\lambda;\rho) \equiv f(x) + \lambda^T h(x) + \rho \parallel h(x) \parallel^2$

- **Penalty parameter** ρ **(not used in computing the step) is updated in rigorously (El-Alem, 1987)**

- **Define**

$$ar \;\; d_c \equiv \mathcal{P}(x_c;\rho_c) - \mathcal{P}(x_c + s_c;\rho_c)$$

and

$$pr \;\; d_c \equiv [f(x_c) - a_c^f(x_c + s_c)] + \rho_c[\parallel h(x_c) \parallel^2 - \parallel a_c^h(x_c + s_c) \parallel^2]$$

- **Update the iterate and** Δ_c **based on** $r = \frac{ared_c}{pred_c}$

AMF: conditions on the trial step

- **To inherit convergence properties from MAESTRO, s_1 and s_2 must satisfy:**
  - **A sufficient decrease condition: s_c is to satisfy a fraction of Cauchy decrease in model k**
  - **A boundedness condition: s_1 to satisfy**

$$\| s_1 \| \leq \mathcal{K} \| h(x_c) \|$$

  for some constant $\mathcal{K}$ independent of the iterates

- **Both are easily satisfied; another trust-region procedure suffices**

One choice of trial step—constraints (objective analogous)

Given $z_0 = x_c, \Delta_c > 0$, **set** $y_0 = z_0, \delta_0 < \Delta_c, v_0 = 0$

For $j = 0, 1, \ldots$, **while remaining within** Δ_c **do** $\{$

Construct $q_j^h(y_j + p) \equiv \| a_c^h(y_j) + \nabla a_c^h(y_j)^T p \|^2$

Find an approximate solution p_j **to**

$$\begin{aligned} &\text{minimize} && q_j^h(y_j + p) \\ &\text{subject to} && \| p \| \leq \delta_j \\ & && \| y_j + p \| \leq \Delta_c \end{aligned}$$

that satisfies FCD for $\| a_c^h \|^2$ **from** y_j

Compute $r = \dfrac{\| a_c^h(y_j) \|^2 - \| a_c^h(y_j + p_j) \|^2}{\| a_c^h(y_j) \|^2 - q_j^h(y_j + p_j)}$

Evaluate p_j **and update** y_j **and** δ_j

Set $v_{j+1} = v_j + (y_{j+1} - y_j)$

$\}$

Set $s_1 = v_j$

Convergence properties

- **Theoretical:**
  - **MAESTRO assumptions: smoothness and boundedness, full rank for the gradients of constraints and their models, sufficient decrease and boundedness for the substeps**
  - **AMF assumptions: consistency conditions and uniform boundedness of the Hessian approximations**
  - **Result: first-order convergence to a critical point of the high-fidelity problem**
- **Practical:**
  - **Enforce compatibility conditions**
  - **Actual performance will depend on the predictive properties of the model; very problem-dependent**

Preliminary Numerical Results

- **Initial testing on Hock and Schittkowski problems and MDOB Test Suite problems; notion of variable-fidelity models not well defined**
- **Now demonstrating feasibility on single-discipline, aerodynamic optimization problems**
- **Variable-fidelity models represented by a single model evaluated of meshes of varying refinement**
- **Computational experiment:**
  - **Single-fidelity problems solved with a well-known optimizers**
  - **Single-fidelity problems solved with a research implementation of MAESTRO, without AMF**
  - **Variable-fidelity problem solved with MAESTRO-based AMF**

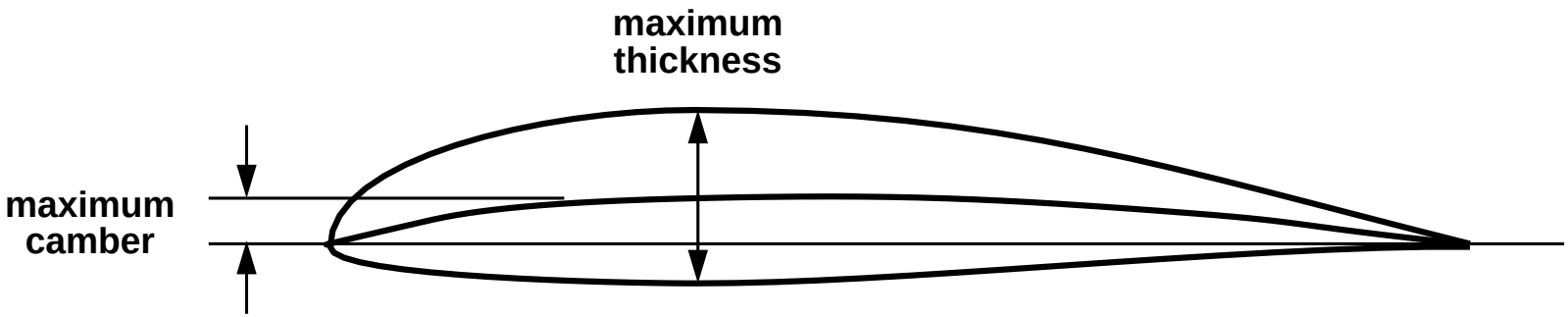
Preliminary Numerical Results: computational details

- **Consistency conditions:**
  - **Enforced only at "major" iterates**
  - **Can be relaxed**
  - **Are easily enforced (Chang et al. '93):**
    - **Given $f_{hi}(x)$ and $f_{lo}(x)$, define $\beta(x) \equiv \frac{f_{hi}(x)}{f_{lo}(x)}$**
    - **Given x_c, build $\beta_c(x) = \beta(x_c) + \nabla\beta(x_c)^T(x - x_c)$**
    - **Then $a_c(x) = \beta_c(x) f_{lo}(x)$ satisfies the consistency conditions**
- **Inequality constraints handled by squared slacks**

Preliminary Numerical Results: 2D Airfoil Optimization

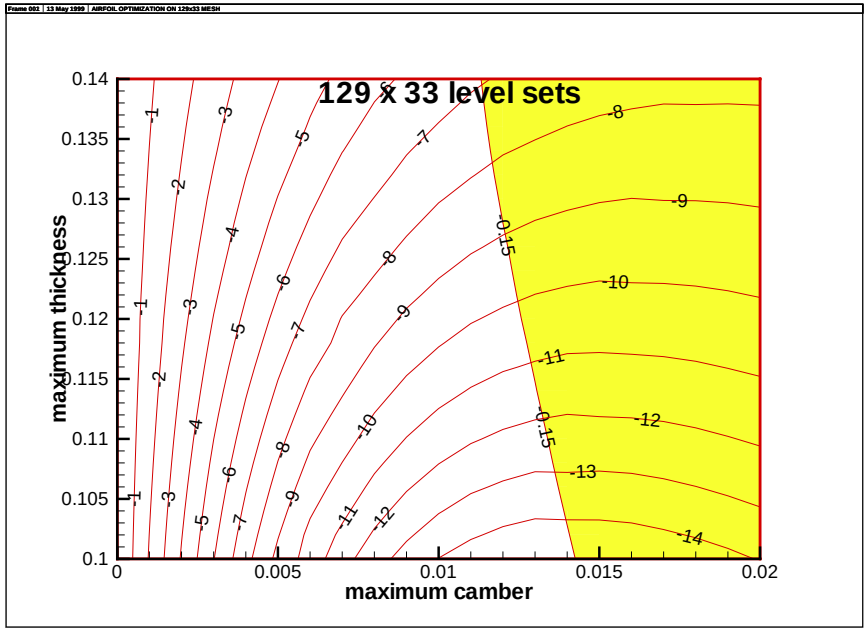
Problem formulated and assembled by L.L. Green

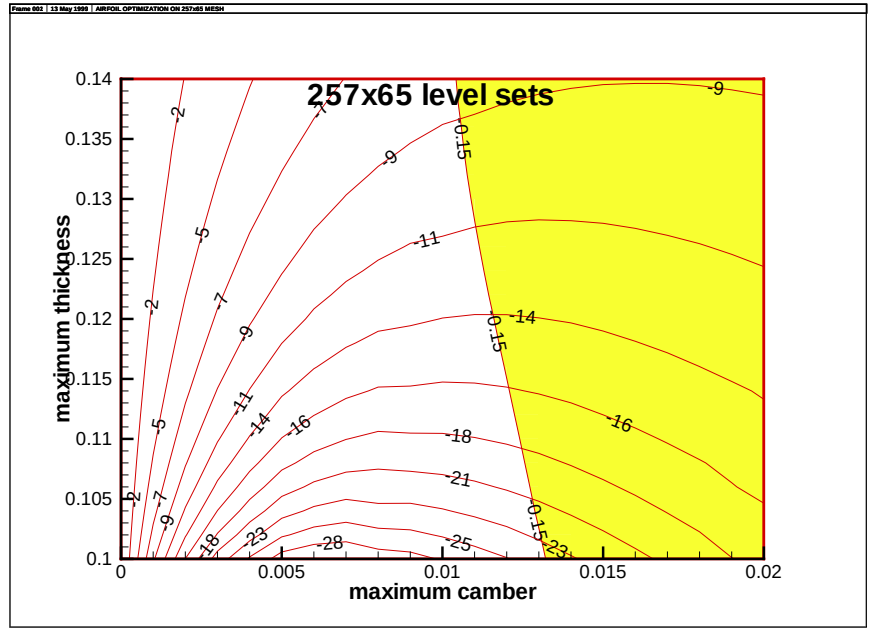
- **Objective:** $-\frac{L}{D}$



- **Constraints:** pitching moment
- **Design variables:** maximum camber, maximum thickness
- **Analysis:** Euler (NS/Euler code FLOMG, Swanson, Turkel)
- **Conditions:** subsonic, inviscid flow
- **Levels of fidelity:** analyses on 129x33, 257x65 meshes

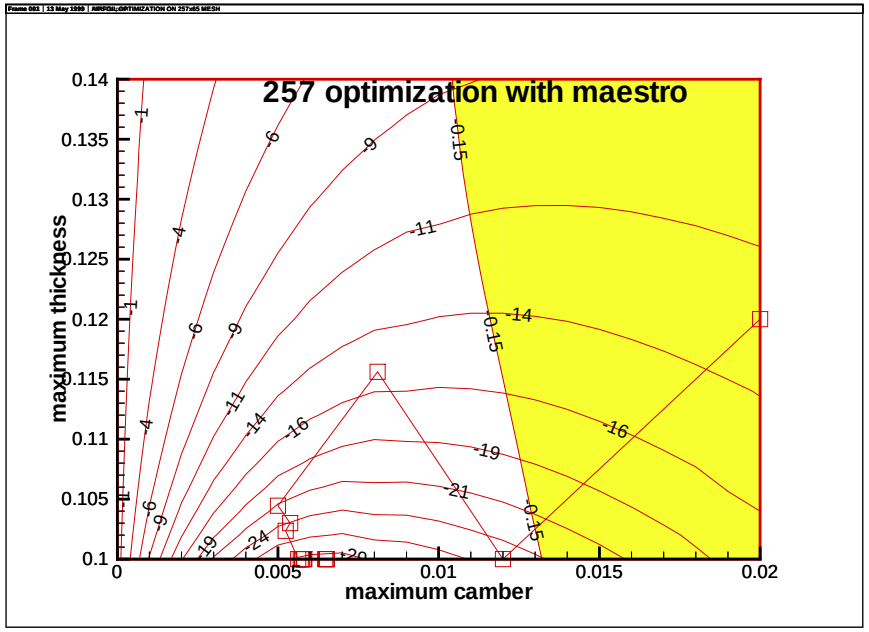
2D Airfoil: Problem Description

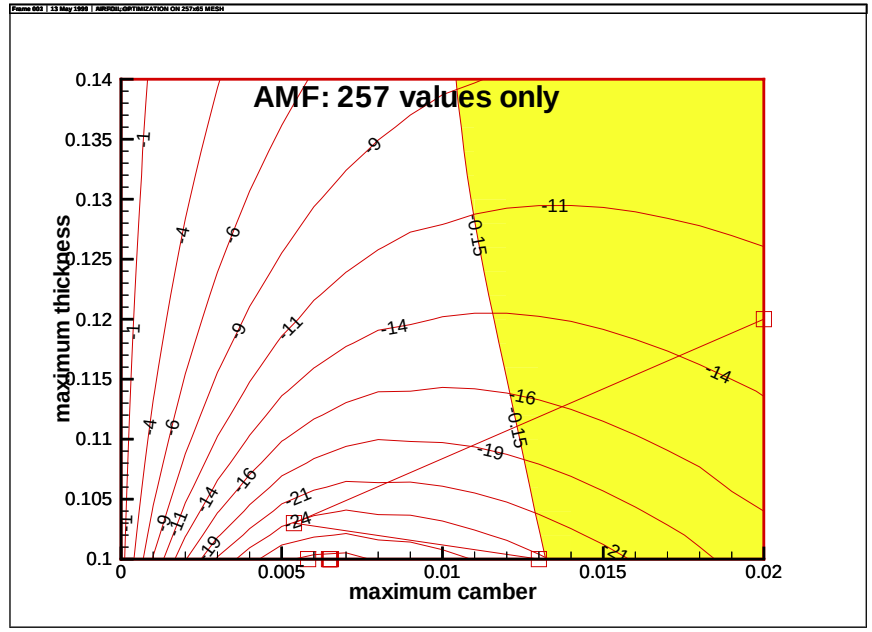




- **Time/analysis on 257x65 mesh = 4 Time/analysis on 129x33 mesh**
- **Approximately 8 min vs 2 min per analysis cold start (free stream conditions)**
- **Restart files are used - neighboring solutions obtained more efficiently**

2D Airfoil: MAESTRO and AMF Results



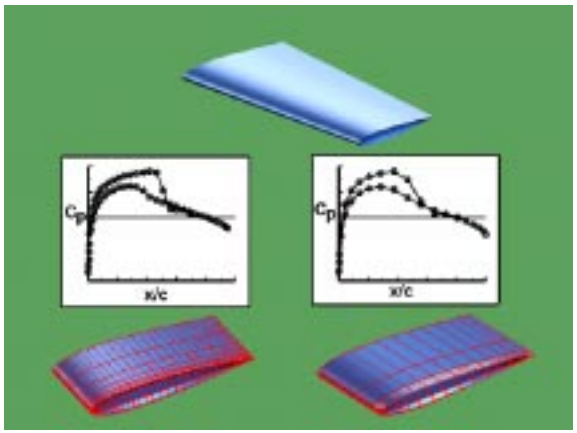


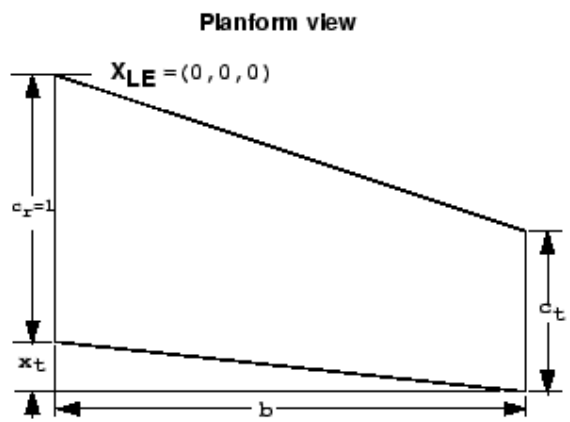
- **Number of iterations:**
  - **MAESTRO: 257 mesh alone - 34**
  - **AMF: on 129 mesh - 20; on 257 mesh - 9; equivalent 257 mesh - 14**

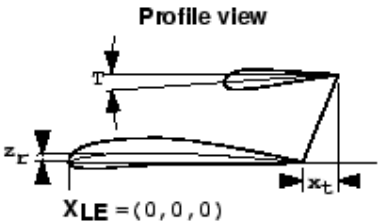
Preliminary Numerical Results: 3D Wing Optimization

Problem formulated and assembled by C.R. Gumbert

- **Objective:** $-\frac{L}{D}$

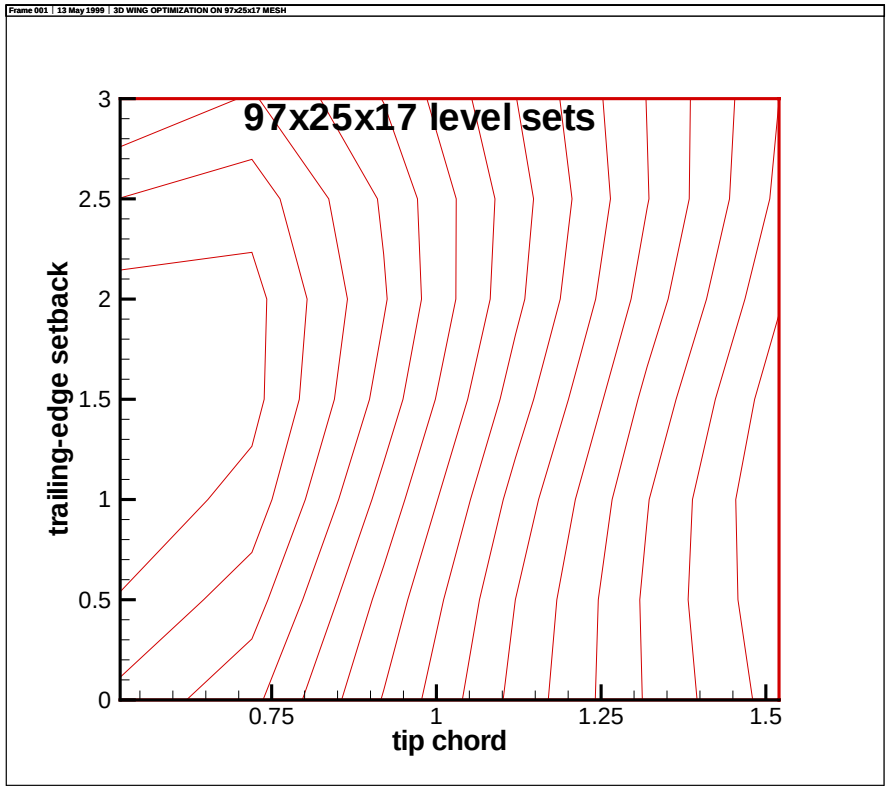


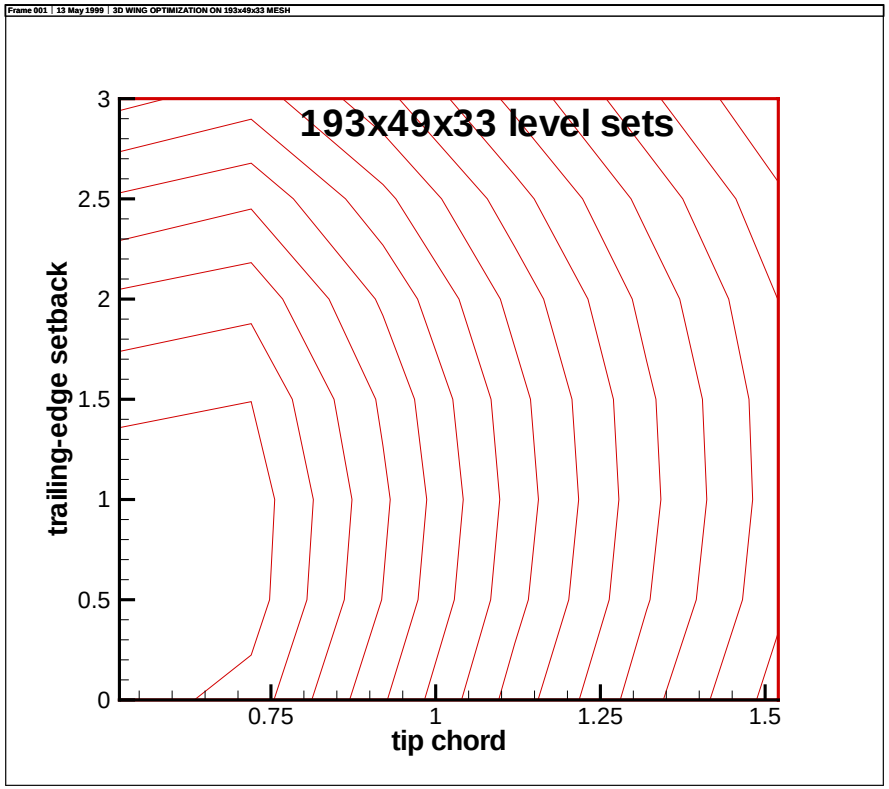




- **Constraints:** $C_L S$ **(total lift);** C_l **(rolling moment);** C_M **(pitching moment)**
- **Design variables: tip chord, trailing edge setback**
- **Analysis: Euler (NS/Euler code CFL3D, NASA LaRC)**
- **Conditions: subsonic, inviscid flow**
- **Levels of fidelity: analyses on 97x25x17, 193x49x33 meshes**

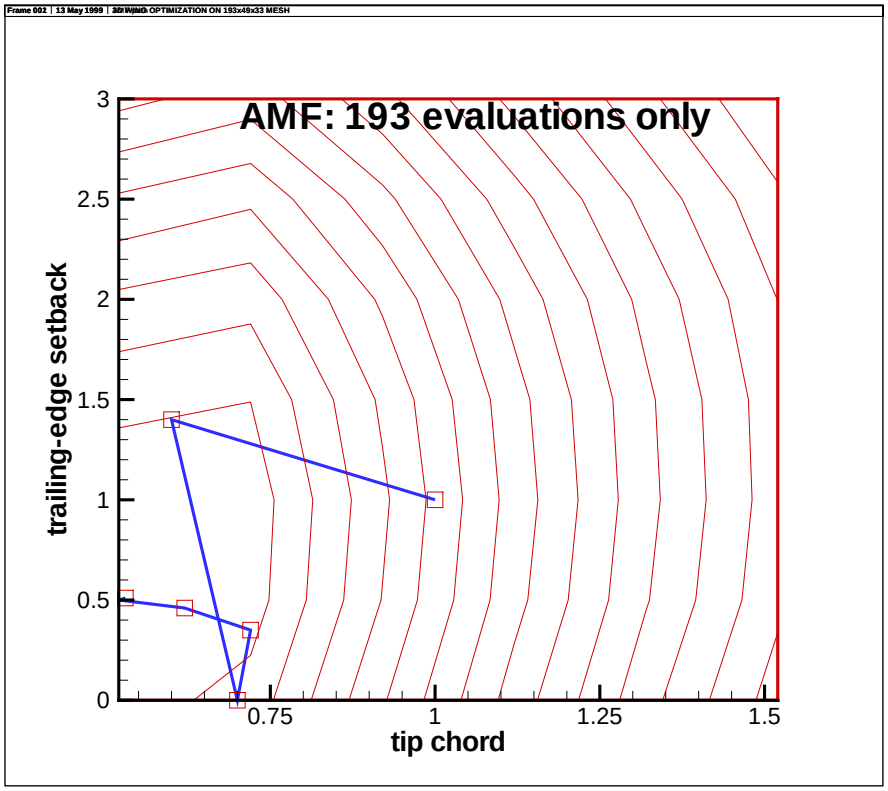
3D Wing: Problem Description





- **Time/analysis on 193x49x33 mesh = 8 Time/analysis on 97x25x17 mesh**
- **Approximately 64 min vs 8 min per analysis, cold start (free stream conditions)**
- **Restart files are used**

3D Wing: MAESTRO and AMF Results



- **Constraints were inactive for this regime**
- **Number of iterations:**
  - **MAESTRO: 97 mesh alone - 17; 193 mesh alone - in progress, expect similar**
  - **AMF: 97 mesh - 17; 193 mesh - 7; equivalent 193 mesh - 9**

Conclusions to-Date

- **Initial numerical results with MAESTRO-based AMF are promising**
- **For models represented by variable mesh sizes, must use consistent families of meshes**
- **Demonstration with engineering analysis codes is difficult**
  - **Standard practice: re-grid at new points; do not take long steps**
  - **We are attempting to use mesh deformation and may take long steps**
  - **Robustness wrt mesh deformation in question**
  - **Results sensitive to analysis convergence**
  - **Analysis and derivatives very sensitive to feasible region (bounds)**
- **Test problem characteristics typical for some classes of problems only**
- **For other problems other models have to be considered**

Currently Under Investigation

- **Strategies for maximizing the use of lower-fidelity models (e.g., using information from *a posteriori* bounds for PDE outputs)**
- **Other CFD problem regimes (e.g., transonic)**
- **A variety of approximations and AMF's**
- **Other model arrangements (variable-fidelity physical, reduced-order models)**
- **Using automatic differentiation for generating derivatives**
- **Demonstrations for multidisciplinary problems**