The Uncertain Future of CFD

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http://fmad-www.larc.nasa.gov/mdob/
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Theme

Quantifying and managing uncertainty in CFD analysis and design is a challenging research area with numerous, non-traditional customers for the CFD community

Contributors

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Outline

- Why bother to account for uncertainty in aerodynamic analysis and design?
- Uncertainty quantification techniques
- Design under uncertainty methods
- Challenges

Who Cares About Aerodynamics Uncertainty?

- NASA space program managers want uncertainty estimates to accompany systems studies that support decisions on next generation reusable launch vehicles
- CFD managers in aerospace companies would make wider use of CFD if results were accompanied by uncertainty estimates
- Structural engineers engaged in reliability-based design need uncertainty distributions
- Controls engineers want aero uncertainty estimates to reduce risk in control law design

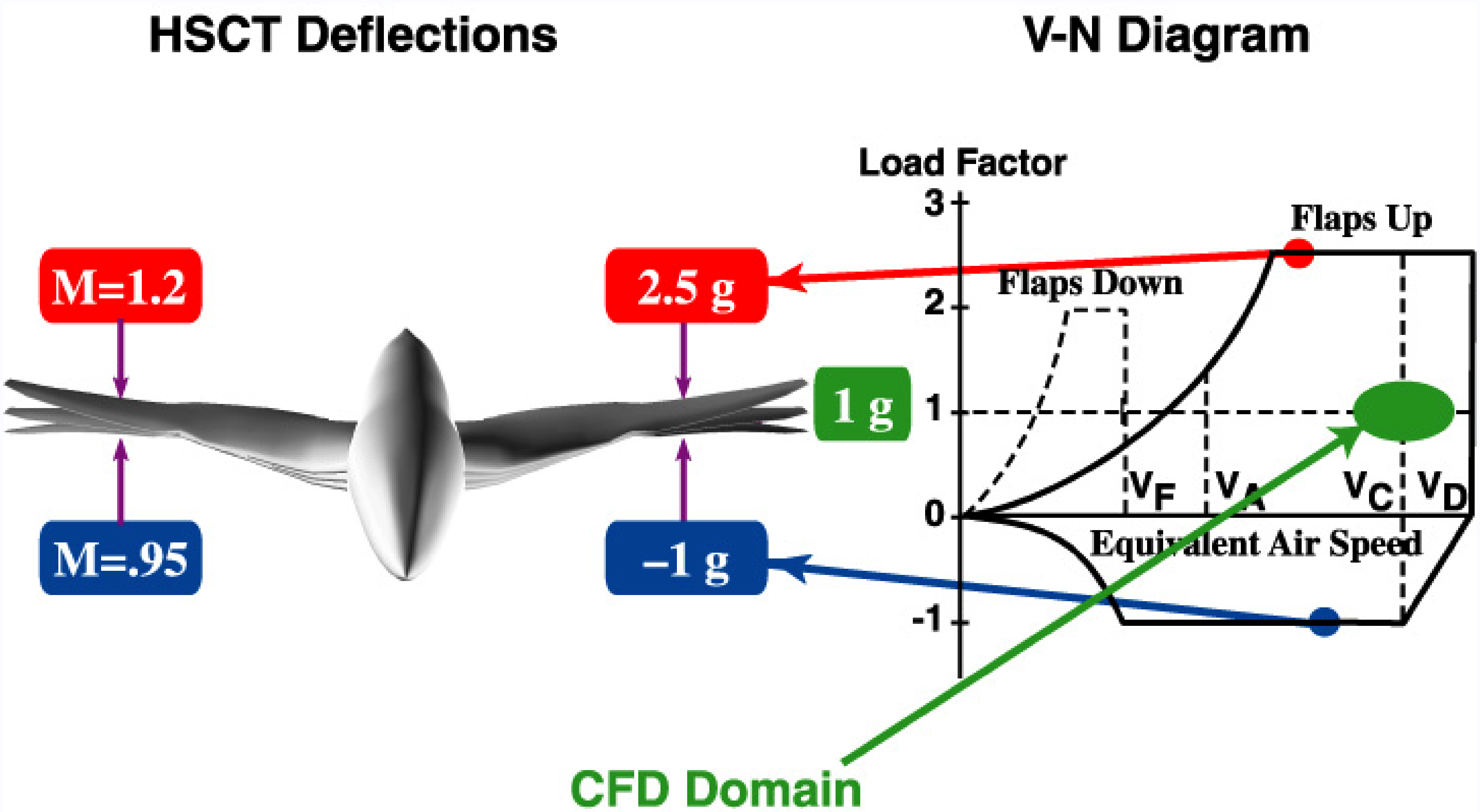
- The DoE ASCI Program has a major thrust in uncertainty quantification

NASA Advanced Space Transportation Goals

(http://www.aero-space.nasa.gov/goals/ast.htm)

- **Access to Space Objective**
  - Reduce the incidence of crew loss by an order of magnitude in 10 years and an additional two orders of magnitude in 25 years
  - Reduce the cost to low-Earth orbit by an order of magnitude in 10 years and another order of magnitude in 25 years
- **Medium/Heavy Payload Challenges**
  - Increase system reliability and performance margins through more robust designs and functional redundancy
  - Optimize system design cycle times
- **Small Payload Challenges**
  - Provide the capability for rapid development and production of highly reliable systems
  - Provide the capability for increased performance margins

CFD Today is Used in a Very Small Region of the Flight Envelop



Aero Performance Uncertainty Targets
(±2 σ)

- **Lift Coefficient**
  - Absolute 0.010
  - Increment 0.005
- **Drag Coefficient**
  - Absolute 0.00010
  - Increment 0.00005
- **Pitching Moment Coefficient**
  - Absolute 0.0010
  - Increment 0.0005

- References

  - Steinle, F., and Stanewsky, E., AGARD-AR-184, November 1982.
  - Carter, E. C., and Pallister, K. C., Chapter 11 in AGARD-CP-429, July 1988.

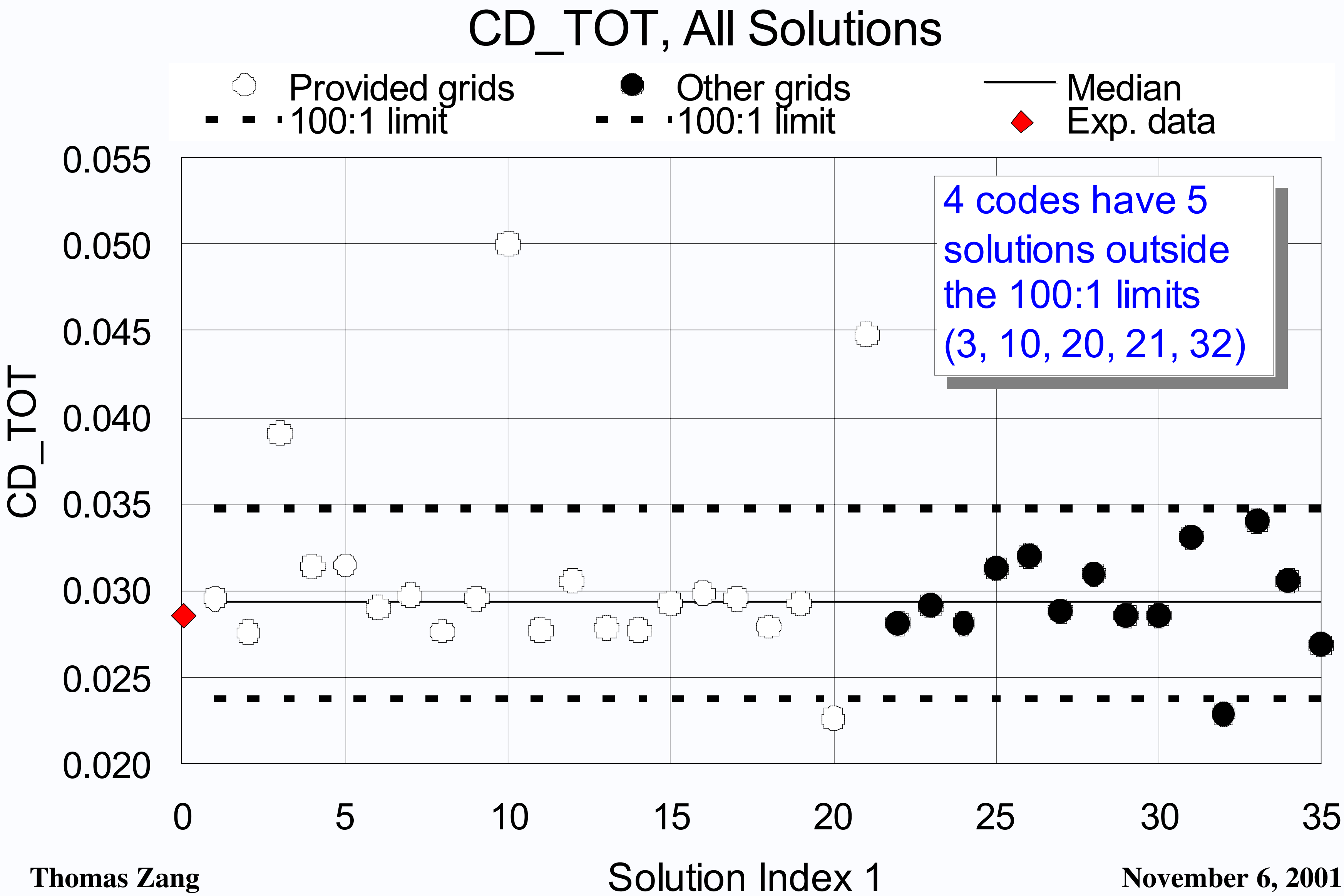
AIAA APA TC Drag Prediction Workshop
June 9-10, 2001

- **14 codes were used:**
  - 7 structured
  - 6 unstructured
  - 1 Cartesian
- **35 solutions for the drag point at C_L=0.5, M=0.75**
  - 17 used Spalart-Allmaras turbulence model
  - 17 used a two-equation turbulence model
  - 1 used Euler-Integral Boundary-Layer method

- **References**
  - Michael J. Hemsch, Statistical Analysis of CFD Solutions from the Drag Prediction Workshop, AIAA Paper 2002-0842
  - http://www.aiaa.org/tc/apa/dragpredworkshop/dpw.html
  - http://ad-www.larc.nasa.gov/tsab/cfdlarc/aiaa-dpw/

Drag at C_L=0.5, M=0.75

from Hemsch's statistical analysis



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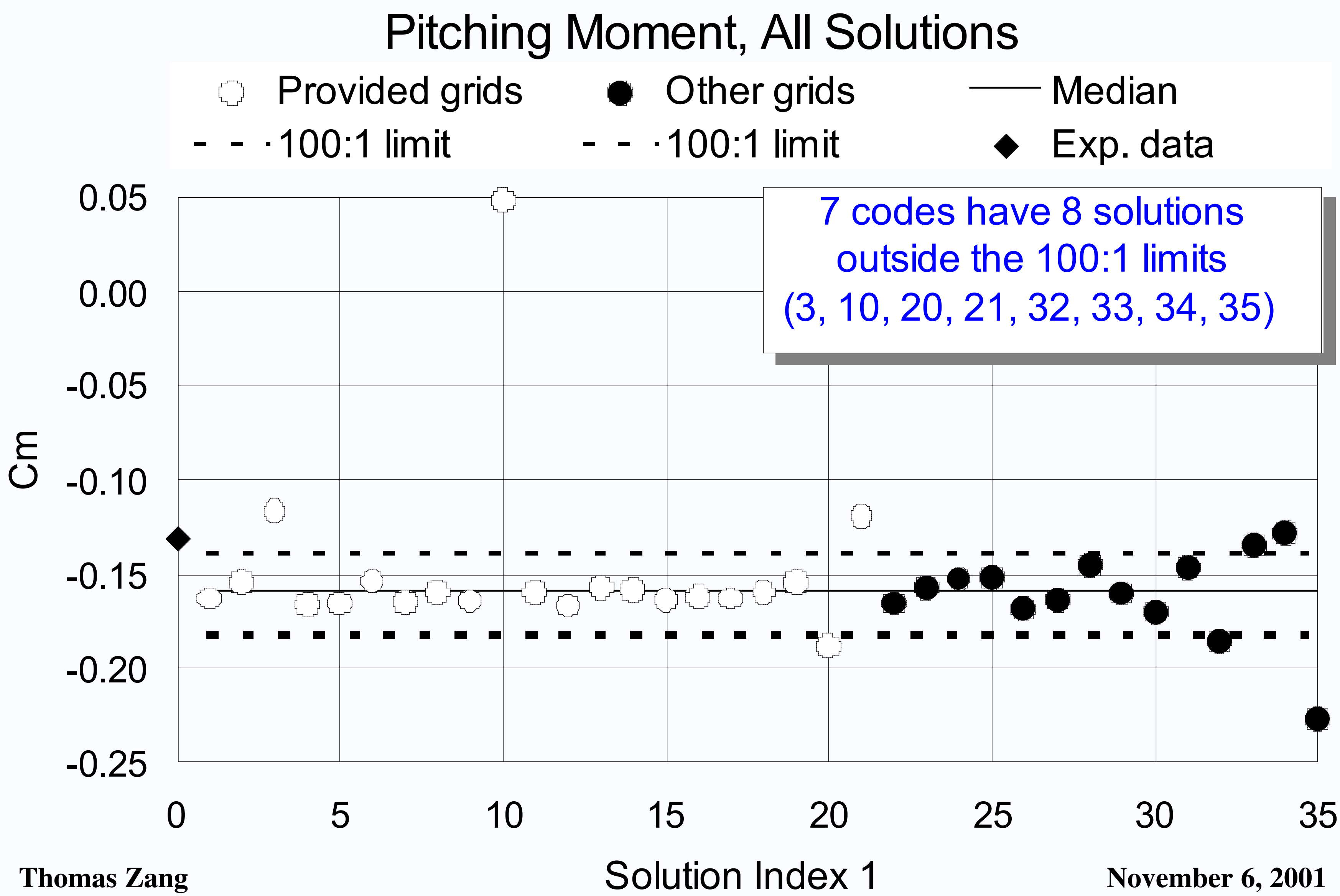
Pitching Moment at C_L=0.5, M=0.75

from Hemsch's statistical analysis



Comparison

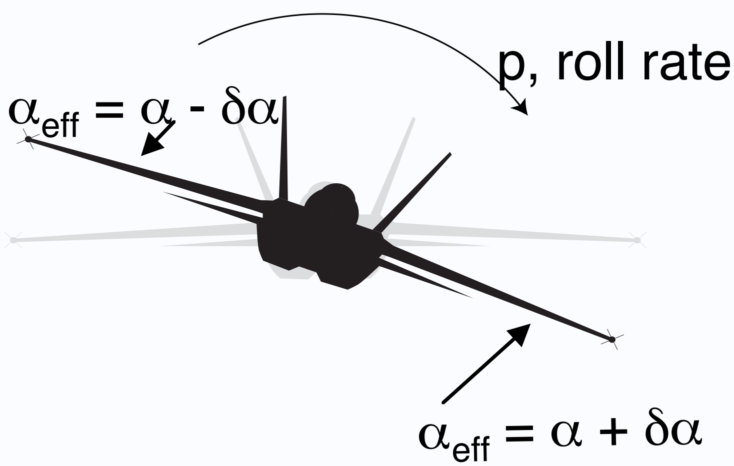
- Drag
  - Goal: 0.0001
  - CFD Dispersion: 0.0021
  - Experimental Dispersion: 0.0004
- Pitching Moment
  - Goal: 0.001
  - CFD Dispersion: 0.008
- Lift (based on scatter in angle of attack)
  - Goal: 0.01
  - CFD Dispersion: 0.005

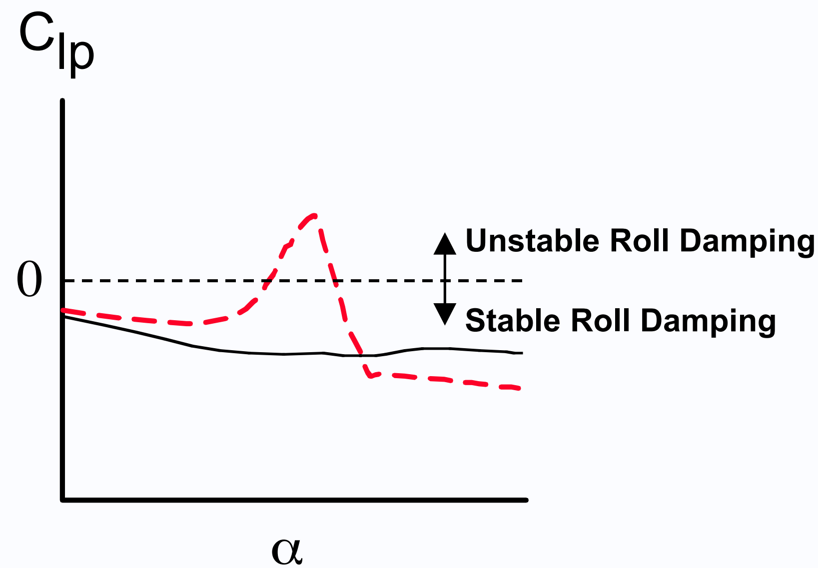
- Current uncertainties on CFD predictions at cruise exceed the goal by a least a factor of 10
- In the parlance of Statistical Process Control, CFD as practiced today is a process that is out of control
- There is a clear need for approaches to managing the CFD process to control uncertainties

Roll Damping Example

- **Simply knowing the sign of C_{lp} with confidence would be very valuable**

C_{lp}: derivative of rolling moment (C_l) with respect to roll rate p





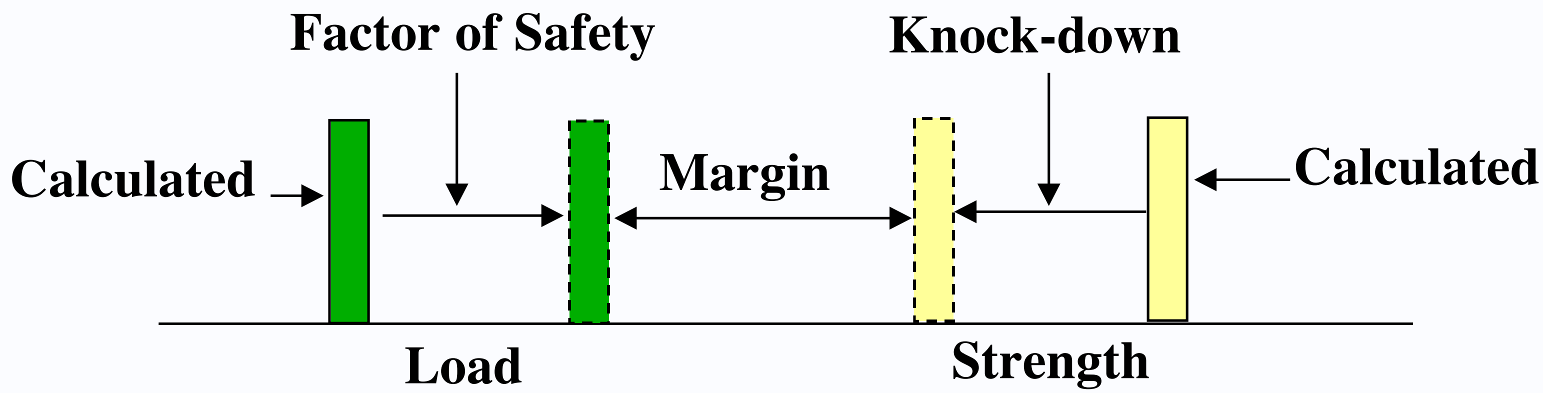
With negative roll damping, down-going wing experiences loss of lift, causing a “propelling” motion

Structural Loads Example

- The critical load cases (those which have the most impact on the structural design) are usually at the edge of the flight envelop
- The accuracy requirements for CFD loads predictions are nowhere near as stringent as those for cruise performance
- The emerging probabilistic structural design methods require probability densities of loads

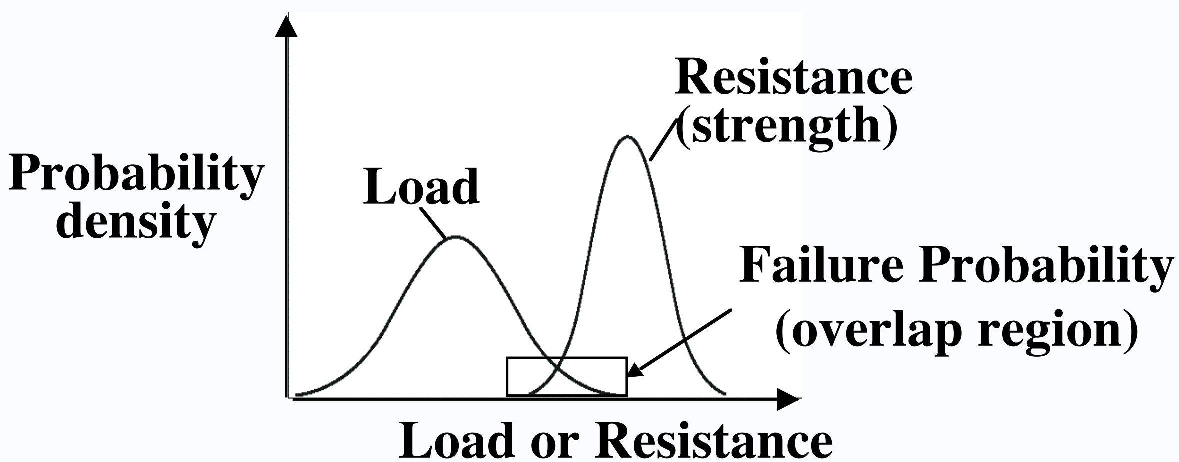
Structural Design Approaches

Factor of Safety Approach



Aero Tools & Data   Structures Tools & Data

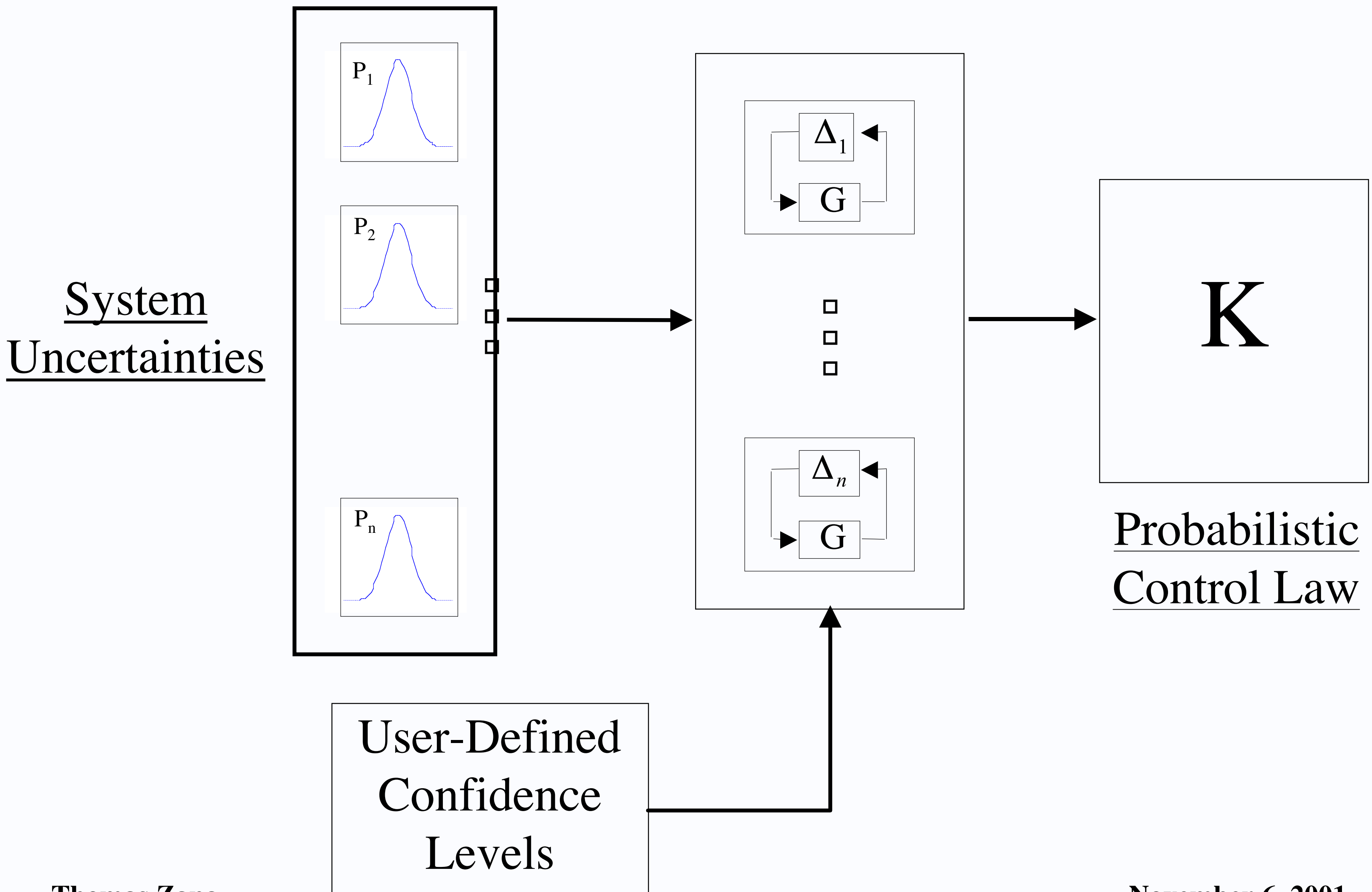
Probabilistic Approach



Stochastic Control Laws Example

- Robust control design (H_∞ control) as developed in the 1970s & 1980s relies solely on bounds for the uncertain parameters
- The goal of current stochastic control law research is to develop control law design methods that exploit probability densities for the uncertain parameters
- The control law designers need probability densities for the uncertain aerodynamics parameters

Robust Control Synthesis



Comments

- CFD is not used in the vast majority of the flight envelop
- The lack of quantitative information on the uncertainty of the CFD results is a contributing factor
- The CFD community appears fixated on quantifying discretization error to the detriment of quantifying other sources of uncertainty
- The challenges lie in quantifying the source of uncertainties and in propagating those uncertainties efficiently through to the “system” level
  - uncertainty sources internal to the code
  - uncertainty sources input to the code

Sources of Uncertainty
(Oberkampf & Blottner, AIAA J., 5/98)

- Physical Models
- Auxiliary Physical Models
- Boundary Conditions
- Initial Conditions
- Discretization and Solution
- Round-Off Error
- Programmer and User Error

Sources of Uncertainty 2
(Oberkampf & Blottner, AIAA J., 5/98)

- **Physical Models**
  - Inviscid Flow
  - Viscous Flow
  - Incompressible Flow
  - Chemically Reacting Gas
  - Transitional/Turbulent Flow
- **Auxiliary Physical Models**
  - Equation of State
  - Thermodynamic Properties
  - Transport Properties
  - Chemical Models, Rates
  - Turbulence Model

Sources of Uncertainty 3

(Oberkampf & Blottner, AIAA J., 5/98)

- **Boundary Conditions**
  - Wall, e.g., roughness
  - Open, e.g., far-field
  - Free Surface
  - Geometry Representation
- **Initial Conditions**
- **Discretization and Solution**
  - Truncation error (spatial and temporal)
  - Iterative convergence error
- **Round-Off Error**
- **Programmer and User Error**

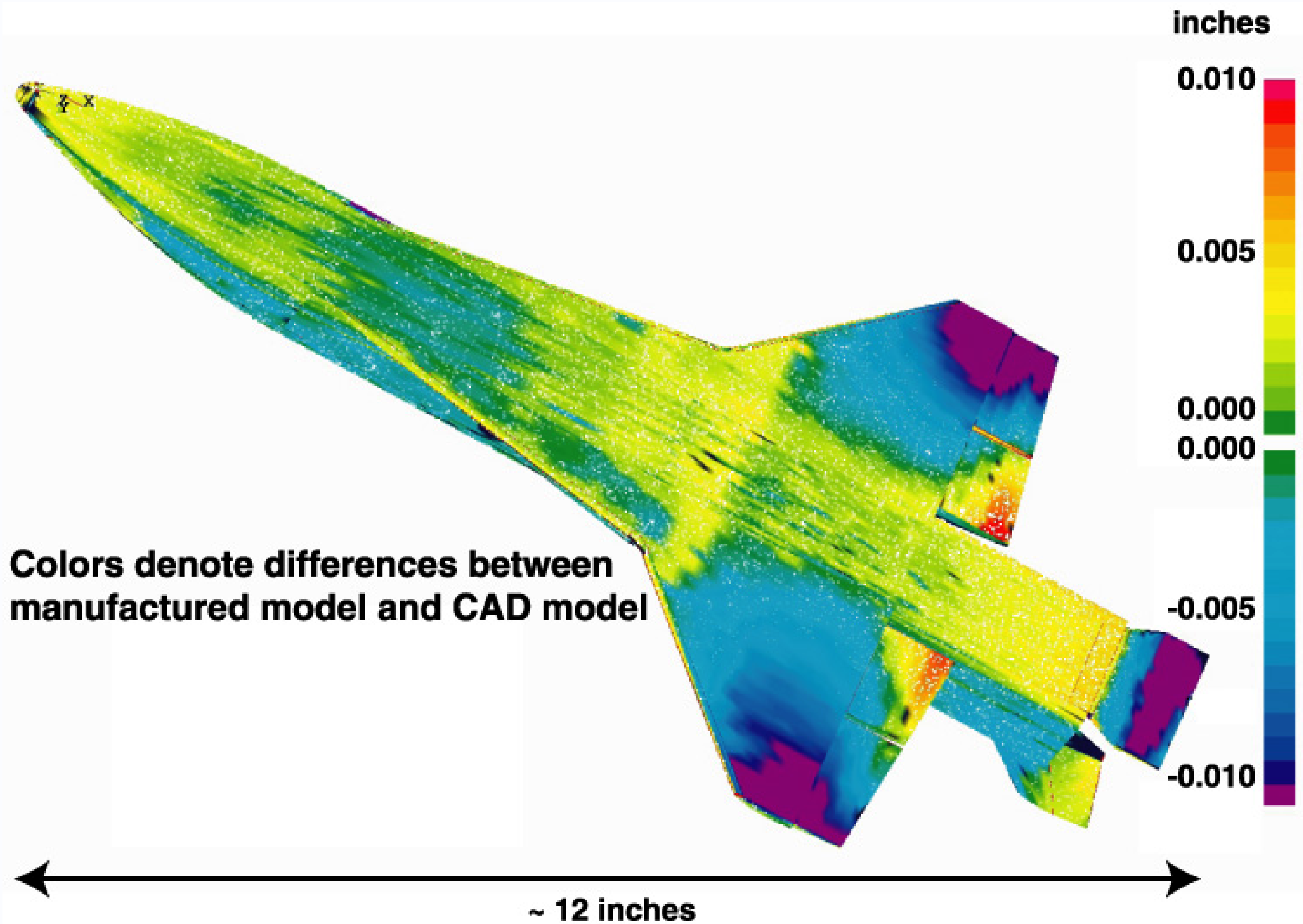
Types of Uncertainty

- *Variability*
  - **the inherent variation associated with the physical system or the environment under consideration**
- *Uncertainty*
  - **a potential deficiency in any phase or activity of the modeling process that is due to lack of knowledge**
- *Error*
  - **a recognizable deficiency in any phase or activity of modeling and simulation that is not due to lack of knowledge**
  - **an error may be either an *acknowledged error* or an *unacknowledged error***

- **Reference**
  - **Oberkampf, Diegert, Alvin and Rutherford, Variability, Uncertainty, and Error in Computational Simulation , ASME-HTD-Vol. 357-2, 1998**

Manufacturing Variability Example

Stereolithographic Measurements of X34 Wind Tunnel Model



Uncertainty Propagation

- Uncertainty propagation deals with estimating the uncertainty in a code's output due to the variabilities, uncertainties and errors in a code's input
- We'll focus on this issue in the middle part of this presentation

Uncertainty Propagation Techniques

- **Interval Analysis**
- **Fuzzy Sets**
- **Sensitivity Estimates**
- ***Moment Methods (e.g., FOSM, SOSM)***
- **Simulation Methods (e.g. Monte Carlo)**
- **Stochastic Finite Elements (Ghanem) & Polynomial Chaos (Karniadakis)**

- **Reference**
  - **Robert Walters, Uncertainty Analysis for Fluid Mechanics with Applications, ICASE Report, in press**

1st and 2nd- Order Taylor Series Approximations for Output F(b)

First-Order:
(FO)

$$\mathbf{F}(\mathbf{b}) = \mathbf{F}(\overline{\mathbf{b}}) + \sum_{i=1}^{n} \frac{\partial \mathbf{F}}{\partial b_i}(b_i - \overline{b}_i)$$

Second-Order:
(SO)

$$\mathbf{F}(\mathbf{b}) = \mathbf{F}(\overline{\mathbf{b}}) + \sum_{i=1}^{n} \frac{\partial \mathbf{F}}{\partial b_i}(b_i - \overline{b}_i) + $$

$$+ \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{\partial^2 \mathbf{F}}{\partial b_i \partial b_j}(b_i - \overline{b}_i)(b_j - \overline{b}_j)$$

where all derivatives are evaluated at the mean values, $\overline{\mathbf{b}}$.

- **Note that efficient first- and second-derivatives are needed from CFD codes**

Approximate Mean and Variance

FO FM: $$\overline{\mathbf{F}} = \mathbf{F}(\overline{\mathbf{b}})$$

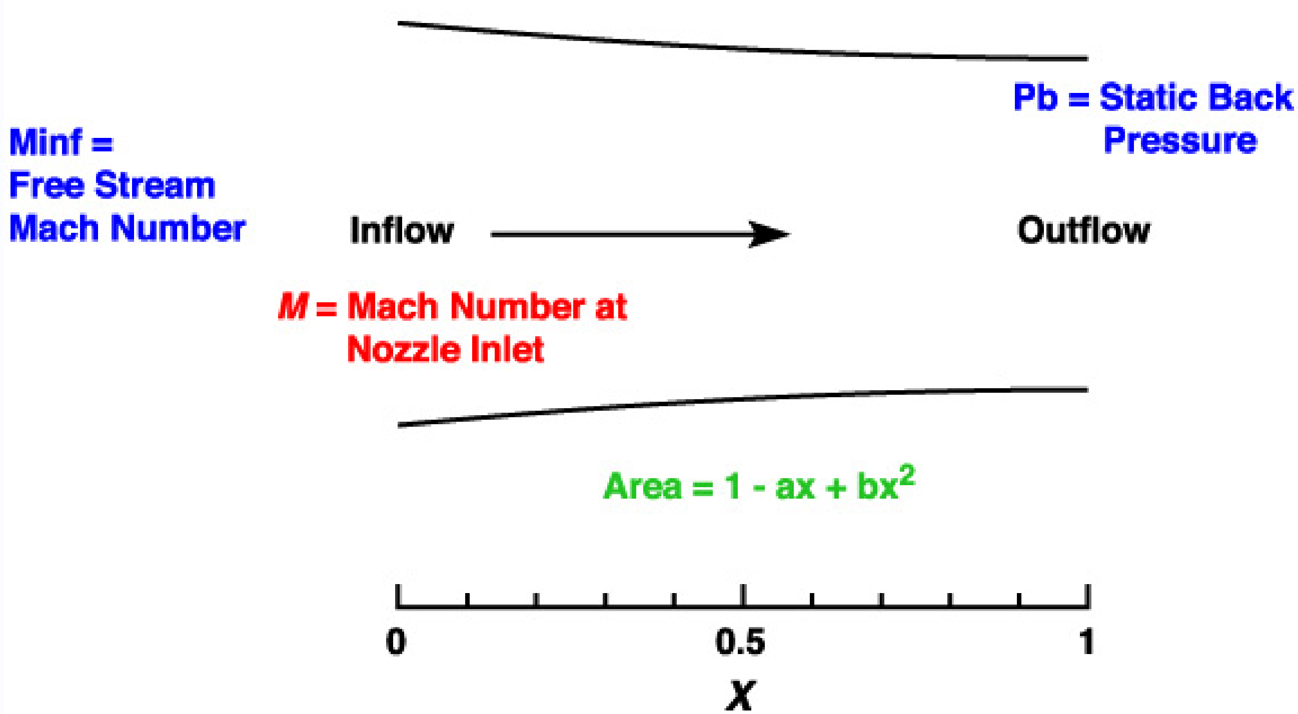
FO SM: $$\sigma_{\mathbf{F}}{}^2 = \sum_{i=1}^{n} \left(\frac{\partial \mathbf{F}}{\partial b_i} \sigma_{b_i} \right)^2$$

SO FM: $$\overline{\mathbf{F}} = \mathbf{F}(\overline{\mathbf{b}}) + \frac{1}{2!} \sum_{i=1}^{n} \frac{\partial^2 \mathbf{F}}{\partial b_i^2} \sigma_{b_i}^2$$

SO SM: $$\sigma_{\mathbf{F}}{}^2 = \sum_{i=1}^{n} \left(\frac{\partial \mathbf{F}}{\partial b_i} \sigma_{b_i} \right)^2 + \frac{1}{2!} \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\frac{\partial^2 \mathbf{F}}{\partial b_i \partial b_j} \sigma_{b_i} \sigma_{b_j} \right)^2$$

Quasi 1-D Euler Problem

Input Random Variables:		CFD Output Function:
Geometric	b={a,b}	F={M}
Flow	b={Minf, Pb}	



Mean and Variance Approximations

FO FM: $$\overline{M} = M(\overline{a}, \overline{b})$$

FO SM: $$\sigma_M{}^2 = \left(\frac{\partial M}{\partial a}\,\sigma_a\right)^2 + \left(\frac{\partial M}{\partial b}\,\sigma_b\right)^2$$

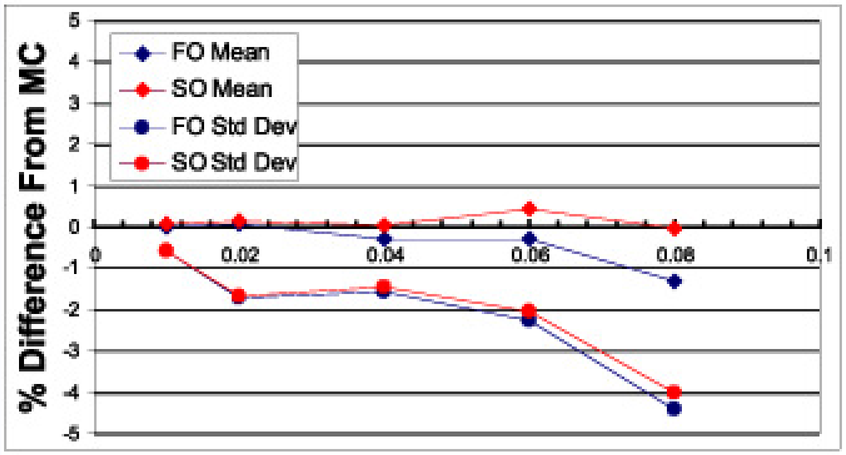
SO FM: $$\overline{M} = M(\overline{a}, \overline{b}) + 0.5\left(\frac{\partial^2 M}{\partial a^2}\right)\sigma_a{}^2 + 0.5\left(\frac{\partial^2 M}{\partial b^2}\right)\sigma_b{}^2$$

SO SM: $$\sigma_M{}^2 = \left(\frac{\partial M}{\partial a}\,\sigma_a\right)^2 + \left(\frac{\partial M}{\partial b}\,\sigma_b\right)^2 + 0.5\left(\frac{\partial^2 M}{\partial a^2}\,\sigma_a{}^2\right)^2 +$$

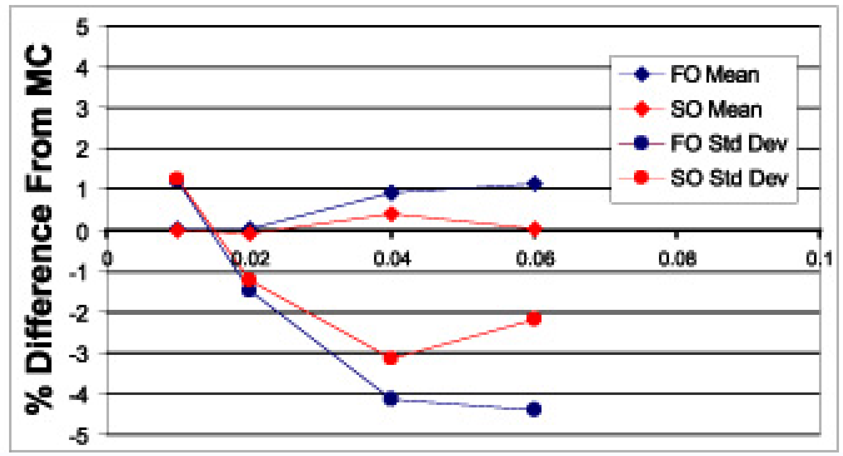
$$+ 0.5\left(\frac{\partial^2 M}{\partial b^2}\,\sigma_b{}^2\right)^2 + \left(\frac{\partial^2 M}{\partial a \partial b}\,\sigma_a\,\sigma_b\right)^2$$

Comparison of Statistical Approximations vs. Monte Carlo Simulation

Geometric Example



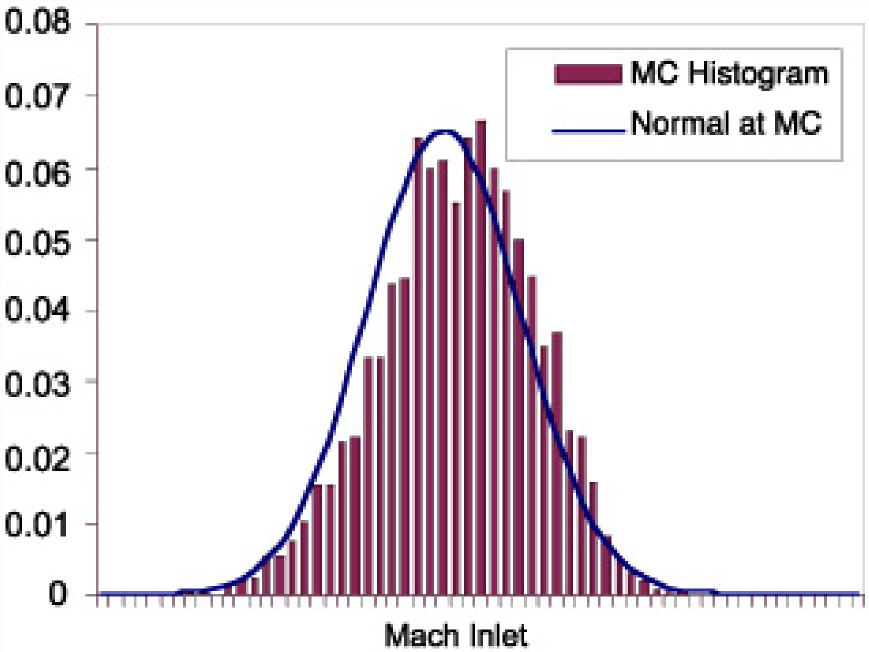
Flow Parameter Example



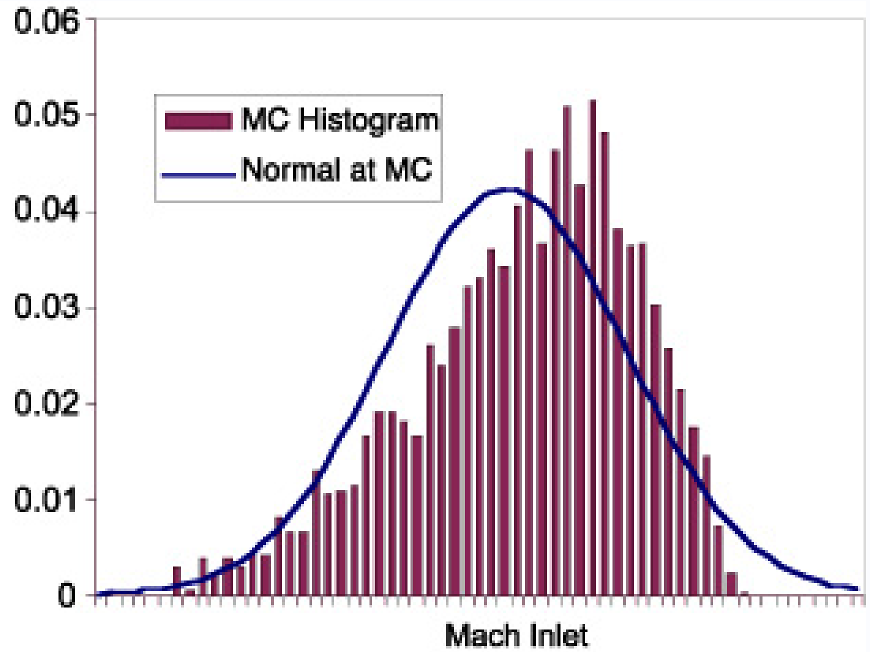
- **For larger values of input parameters, second-order generally gives better predictions**
- **Approximations predict first moment more accurately than second moment**

- **Reference**
  - **Putko, Newman, Taylor & Green, Approach for Uncertainty Propagation and Robust Design in CFD Using Sensitivity Derivatives, AIAA 2001-2528**

Probability Density Functions from Monte Carlo Simulations



$\sigma_{Minf} = \sigma_{Pb} = 0.02$



$\sigma_{Minf} = \sigma_{Pb} = 0.06$

- **The actual Monte Carlo results are compared with a normal distribution using the mean & standard deviation of the Monte Carlo results (graphically indistinguishable from FOSM & SOSM)**
- **The FOSM & SOSM results appear adequate for robust design but not for reliability-based design**

Probabilistic Design Categories

- **Robust Design**
  - a design is sought that is relatively insensitive to small changes in the uncertain quantities
- **Reliability-Based Design**
  - a design is sought that has a probability of failure that is less than some acceptable (invariably small) value

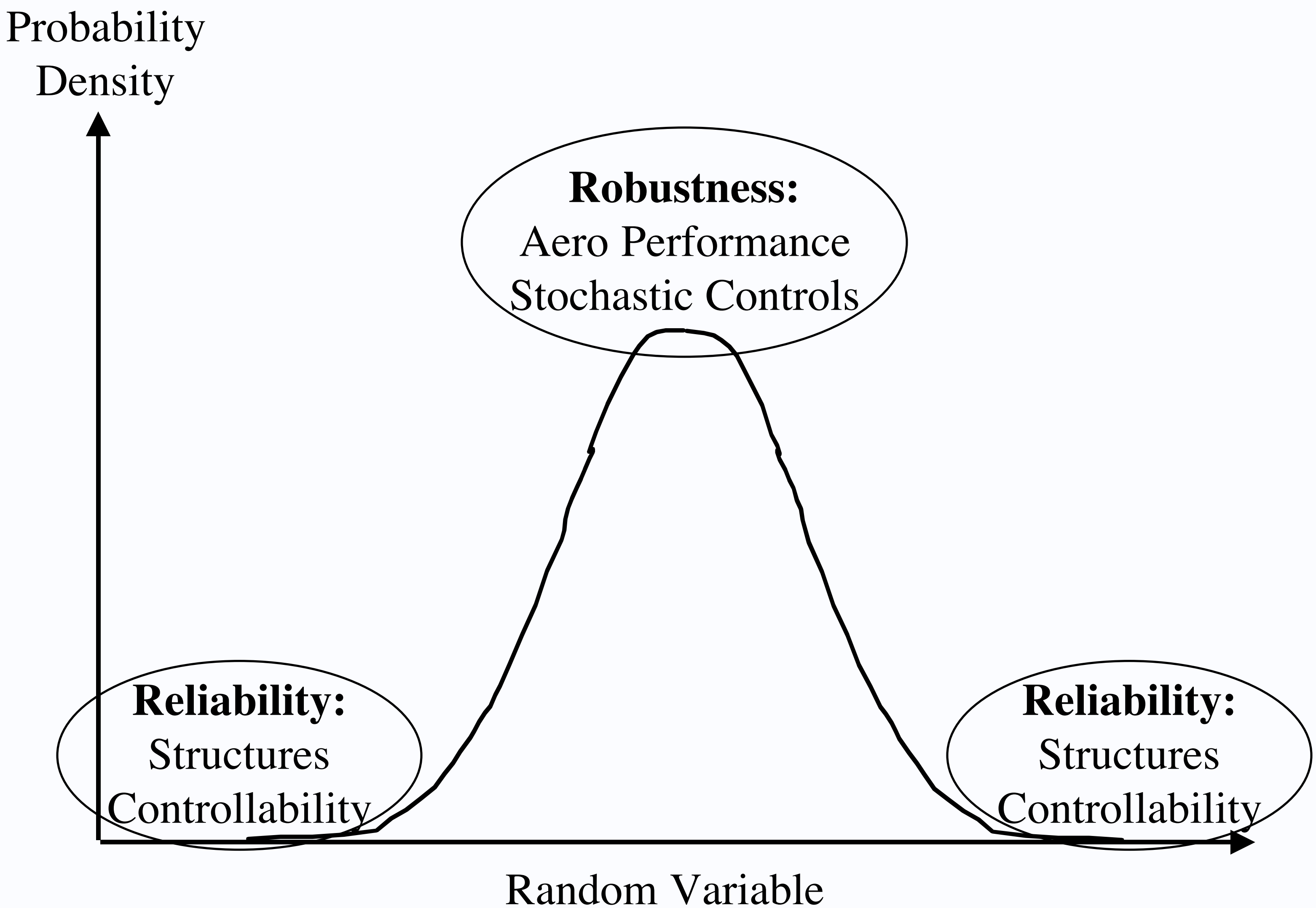
Probabilistic Problem Classification

Impact of Event

	Everyday Fluctuations	Extreme Events
Catastrophe	***No engineering applications***	**Risk analysis Reliability-based design & optimization**
Performance Loss	**Cost-benefit analysis Robust design and optimization**	***Reliability is not an issue***

Frequency of Event

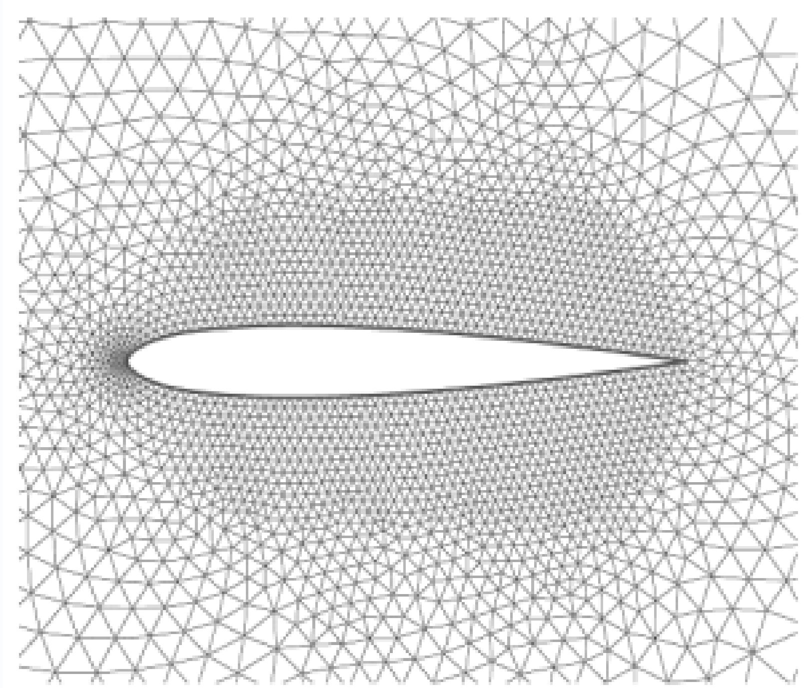
Probability Density vs. Problem Focus



Robust Aerodynamic Shape Optimization

- **Objective**
  - Minimize drag over a range of Mach numbers
  - Limit the number of aerodynamic analyses
- **Design vector *d***
  - angle of attack and 20 box-constrained *y*-coordinates of the control points for the airfoil spline

FUN2D Grid



- **References**
  - Luc Huyse, Solving Problems of Optimization Under Uncertainty As Statistical Decision Problems., AIAA 2001-1519
  - Wu Li, Sharon Padula, and Luc Huyse, Robust Airfoil Optimization to Achieve Consistent Drag Reduction over a Mach Range, ICASE Report No. 2001-22

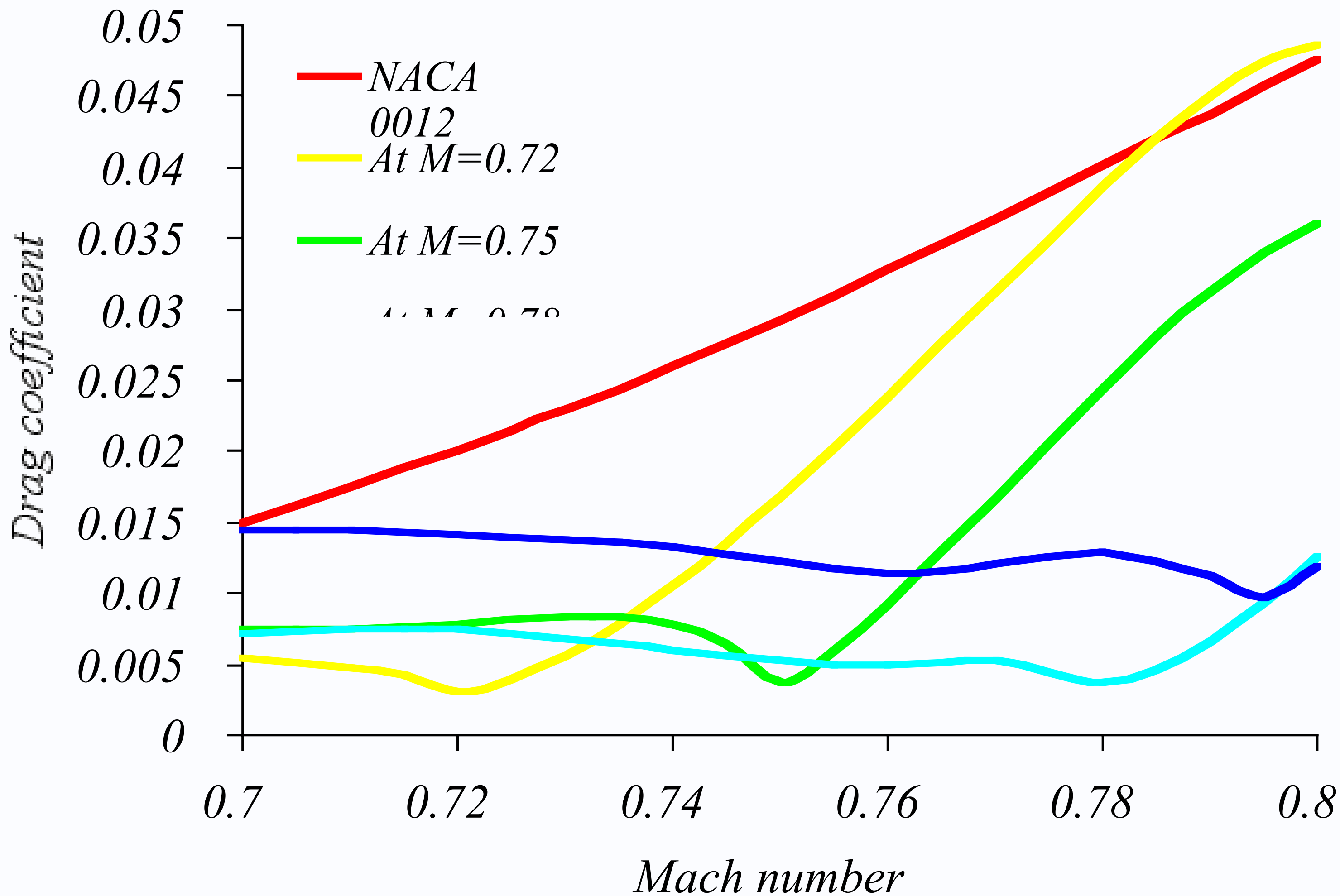
Single Design-Point Optimization

- **The design vector *d* (geometry and angle of attack) is the only variable in the objective**
- **Fix all other model parameters at their design value. We consider only 1 free flow Mach number $\boldsymbol{M = M_{design}}$ (*e.g. average Mach number during cruise stage*):**

$$
\begin{cases}
\min\limits_{d \in D} \quad C_d(d, M_{design}) \\
\text{subject to} \quad C_l(d, M_{design}) \geq C_l^*
\end{cases}
$$

Problems with Single Point Optimization

- **Choice of M_{design} dramatically affects performance**



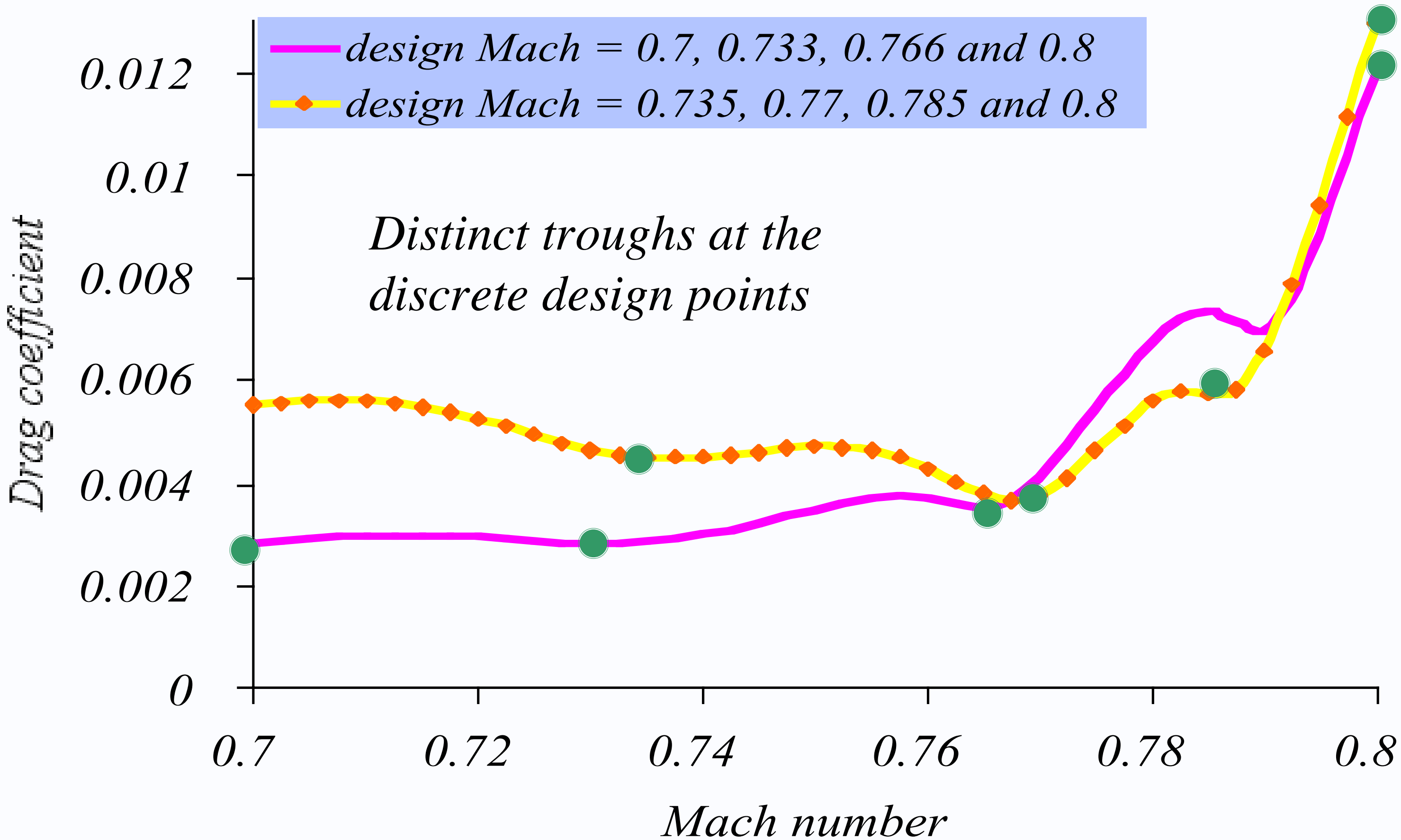
Multi-Point Optimization

- The design vector *d* (geometry and angle of attack) is the only variable in the objective
- Consider multiple design conditions at selected values of the free flow Mach number
- Objective function is a weighted average of all these design conditions

$$\begin{cases} \min\limits_{d \in D} \quad \sum_{i=1}^{n} w_i C_d(d, M_i) \\ \text{subject to} \quad C_l(d, M_i) \geq C_l^* \quad \text{for} \quad i = 1, n \end{cases}$$

Problems with Four-Point Optimization

- **Choice of design conditions affects performance**



Stochastic Optimization

- **Modify the objective to directly incorporate the effects of model uncertainties on the design performance**
- **Highlight 2 methods:**
  - **Expected Value Optimization**
  - **Second-Order Approximate Results**

Mathematical Formulation

- **Minimize the expected value of the drag over the design lifetime:**

$$\min_{d \in D} E_M\big(C_d(d, M)\big) = \min_{d \in D} \int_M C_d(d, M) f_M(M) dM$$

C_d is drag function

d is design vector (geometry, angle of attack)

M is uncertain parameter (Mach number)

f_M is Probability Density Function of Mach number

Application to Airfoil Problem

- **Integrate over the uncertain parameter *M*, compute the expected value of C_d with respect to the free flow Mach number *M*.**
- **Minimize this integrated objective with respect to the design vector *d*.**
- **Actual flight data can be readily incorporated in the probability density function $f_M(M)$**

$$\begin{cases} \min\limits_{d \in D} & \int_M C_d(d, M) f_M(M) dM \\ \text{subject to} & C_l \geq C_l^* \end{cases}$$

SOSM Approximation

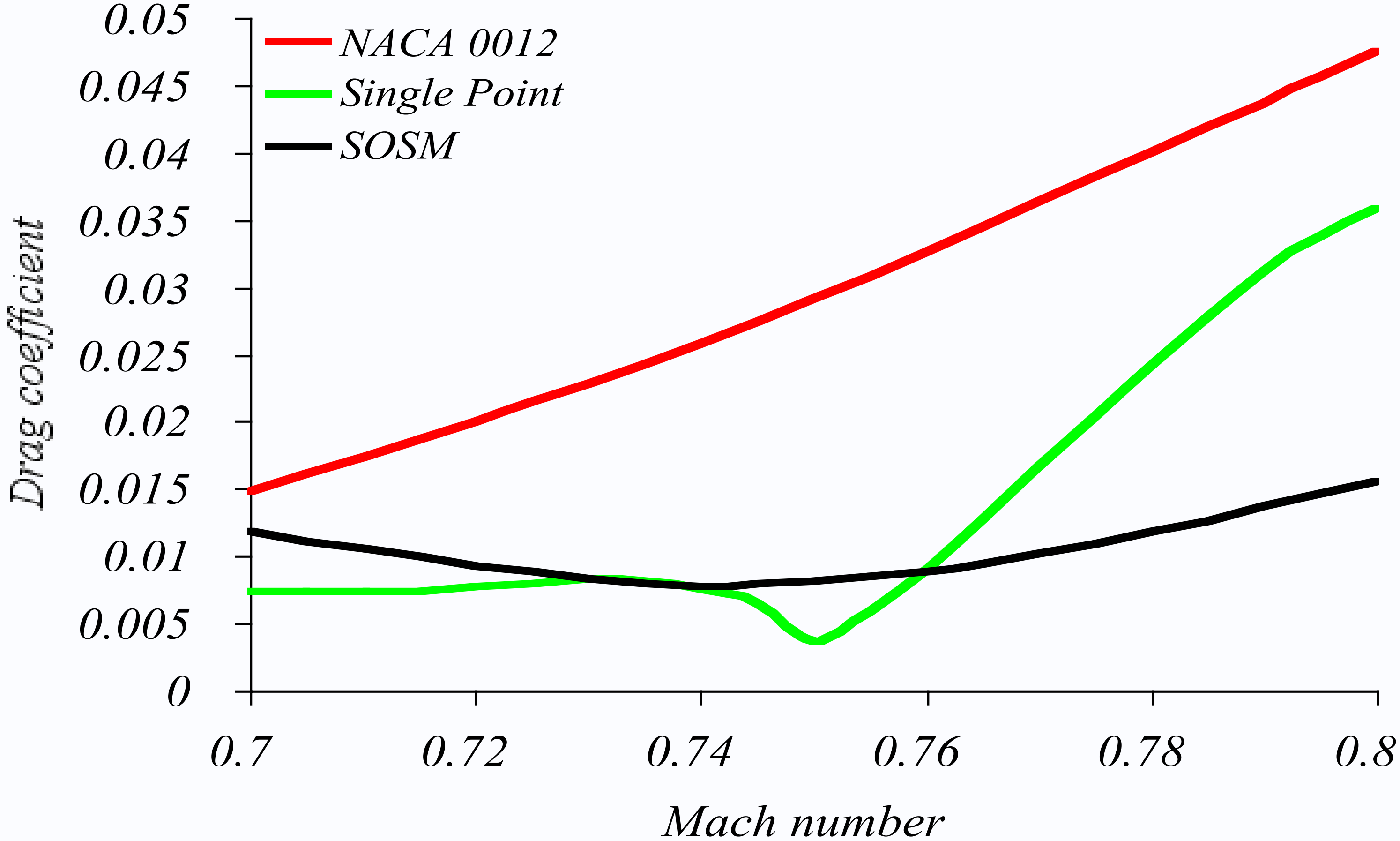
- **Approximate objective by second-order Taylor series expansion about the *mean value of M*, and evaluate the expectation integral analytically**

$$\min_{d \in D} \int_M C_d(d, M) f_M(M) dM \cong$$

$$\min_{d \in D} \left[C_d(d, \overline{M}) + \tfrac{1}{2} Var(M) \left. \frac{\partial^2 C_d}{\partial M^2} \right|_{M = \overline{M}} \right]$$

$$subject\ to: \quad C_l \geq C_l^*$$

Comparison with Single Point Opt.

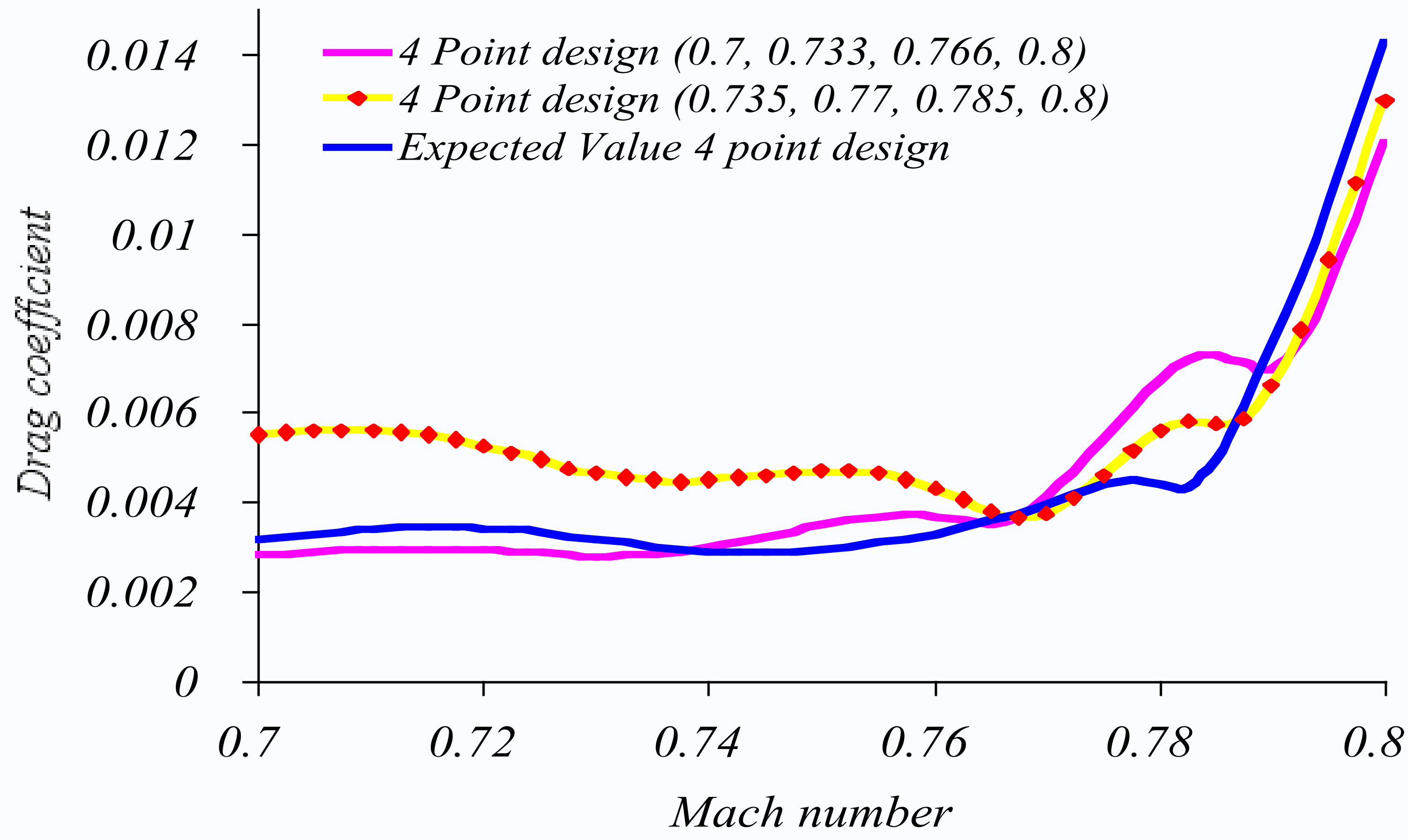


Direct Evaluation of Integral

- Evaluate integral directly using a numerical integration method.
- To avoid over-optimization, make sure you select different integration points for each optimization step.
- We used 4-point integration with random selection of integration points.

Comparison with Multi-Point Optimization

- Expect Value design is independent of arbitrary selection of Mach numbers

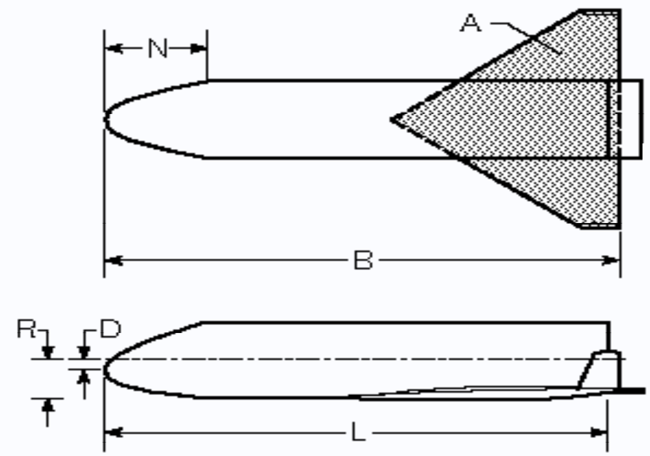


Relative Computational Effort

Optimization Method	1 Random Variable	3 Random Variables
Single-Point	1	1
SOSM(*)	3	7
Expected Value (4pts)	4	64

(*) Less if analytic derivatives are available

Reliability-Based Design Example
Controllability of Reentry Vehicle

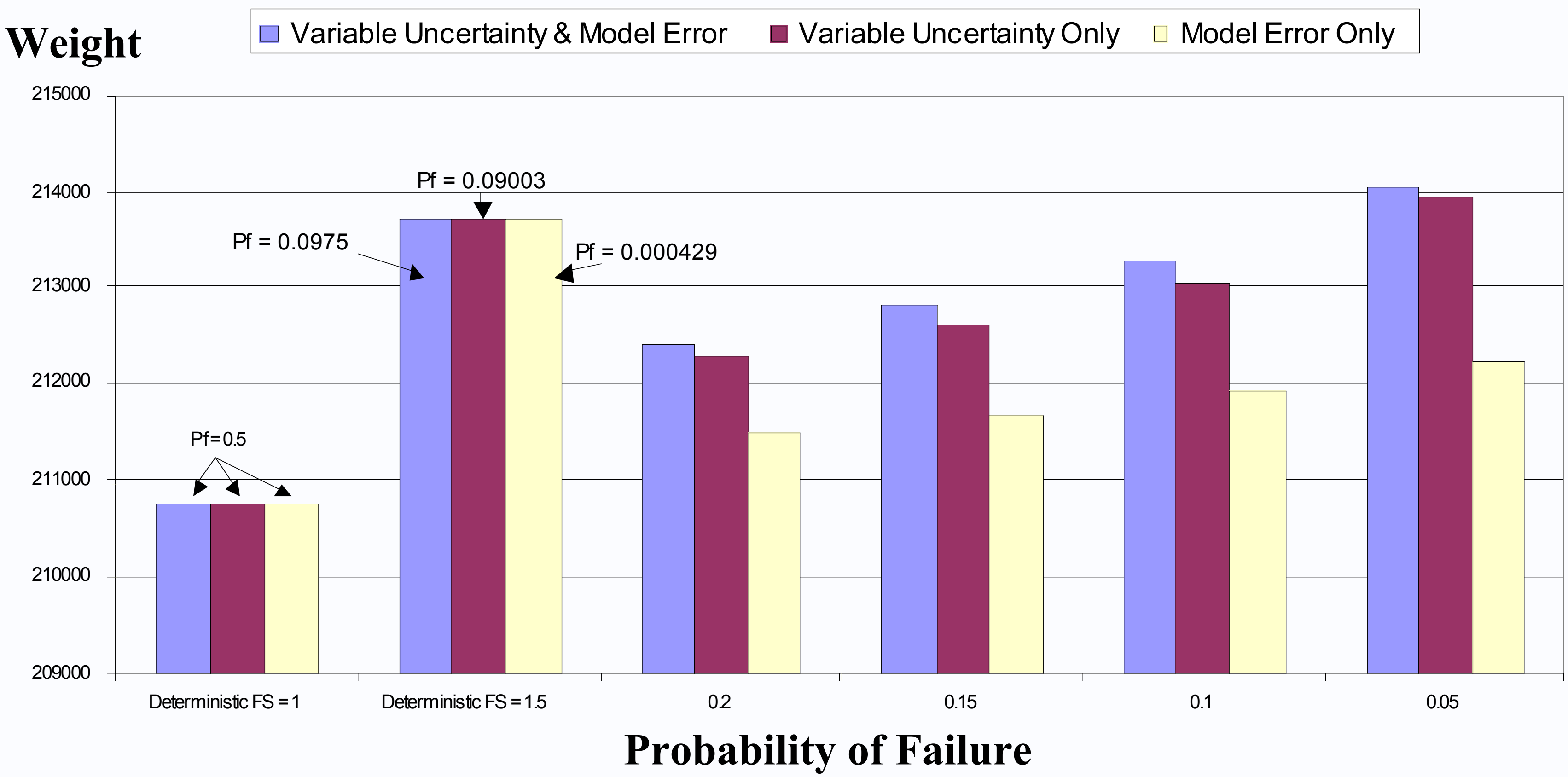


- Objective:
  - **minimize dry weight**
- Design Variables (5):
  - **configuration parameters**
- Constraints (7):
  - **landing speed; hypersonic, supersonic, and subsonic trim and stability levels**
- Disciplines (3):
  - **geometry, aerodynamics, and weights/sizing**
- Probabilistic Formulation:
  - **Minimize *mean* weight such that pitching moment coefficient for 9 scenarios has a low probability (less than 0.1) of failing to be within acceptable bounds [-0.01, 0.01]**

Design Variable	**Range**
Fuselage fineness ratio	4 - 7
Wing area ratio	10 – 20
Tip fin area ratio	0.5 - 3
Ballast wt fraction	0 – 0.4
Mass Ratio	7.75 – 8.25

Reliability-Based Design Results

Minimum Empty Weight w. Probabilistic Controllability Constraints



Some Challenges for CFD Uncertainty Analysis and Design

- Quantification of transition & turbulence modeling uncertainty
- Affordable simulation strategies for CFD
- Statistical process control techniques for CFD
- Uncertainty quantification strategies for strongly nonlinear problems
- Robust design and reliability-based design algorithms tuned to the characteristics of CFD codes
  - iterative solution of nonlinear systems
  - efficient sensitivity derivatives
- Sparse data on uncertainty distributions
- Strategies for predicting flight loads based on computational, wind tunnel and flight test data