

# Aerospace Applications of Optimization under Uncertainty

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More info: <http://mdob.larc.nasa.gov/>

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## NASA Langley Research Center

- Mathematician
- Multidisciplinary Optimization Branch
- Aerospace Systems, Concepts & Analysis Organization



 LaRC Aircraft in Front of Hangar, Building 1244  
NASA Langley Research Center 6/27/1994 Image # EL-1996-00055

# Outline

1. Three examples of optimization under uncertainty
  - Impact Dynamics
  - Coupled Aerodynamics/Structures
  - Airfoil Shape Optimization
2. Lessons Learned
3. Enabling Technologies Identified

# Three Examples of Optimization under Uncertainty

## Characteristics of Test Problems

- **Similarities**
  - Computationally expensive simulations
  - Sensitive to uncertain input parameters
  - Continuous design variables
  - Nonlinear objectives
- **Differences**
  - Number of design variables from 3 - 50
  - Optimize expected value or limit probability of failure
  - Uncertainty quantification (UQ) either home grown or commercial codes
  - Types of uncertainty

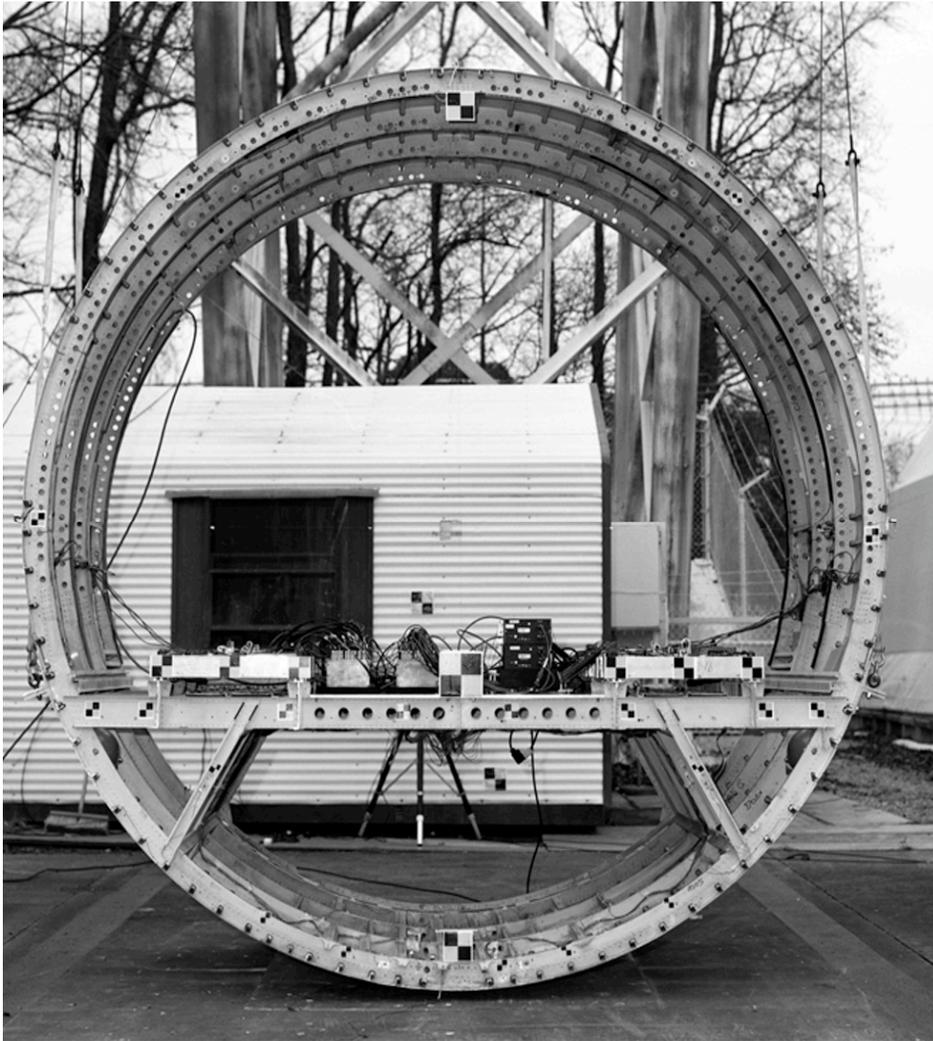
# Outline

1. Three examples of optimization under uncertainty
  - Impact Dynamics
    - Lyle, Padula, Stockwell, “Applications of Probabilistic Analysis to Aircraft Impact Dynamics”, AIAA-2003-1482.
  - Coupled Aerodynamics/Structures
  - Airfoil Shape Optimization
2. Lessons Learned
3. Enabling Technologies Identified

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# Impact Dynamics Example

Can we design a vehicle for crashworthiness?



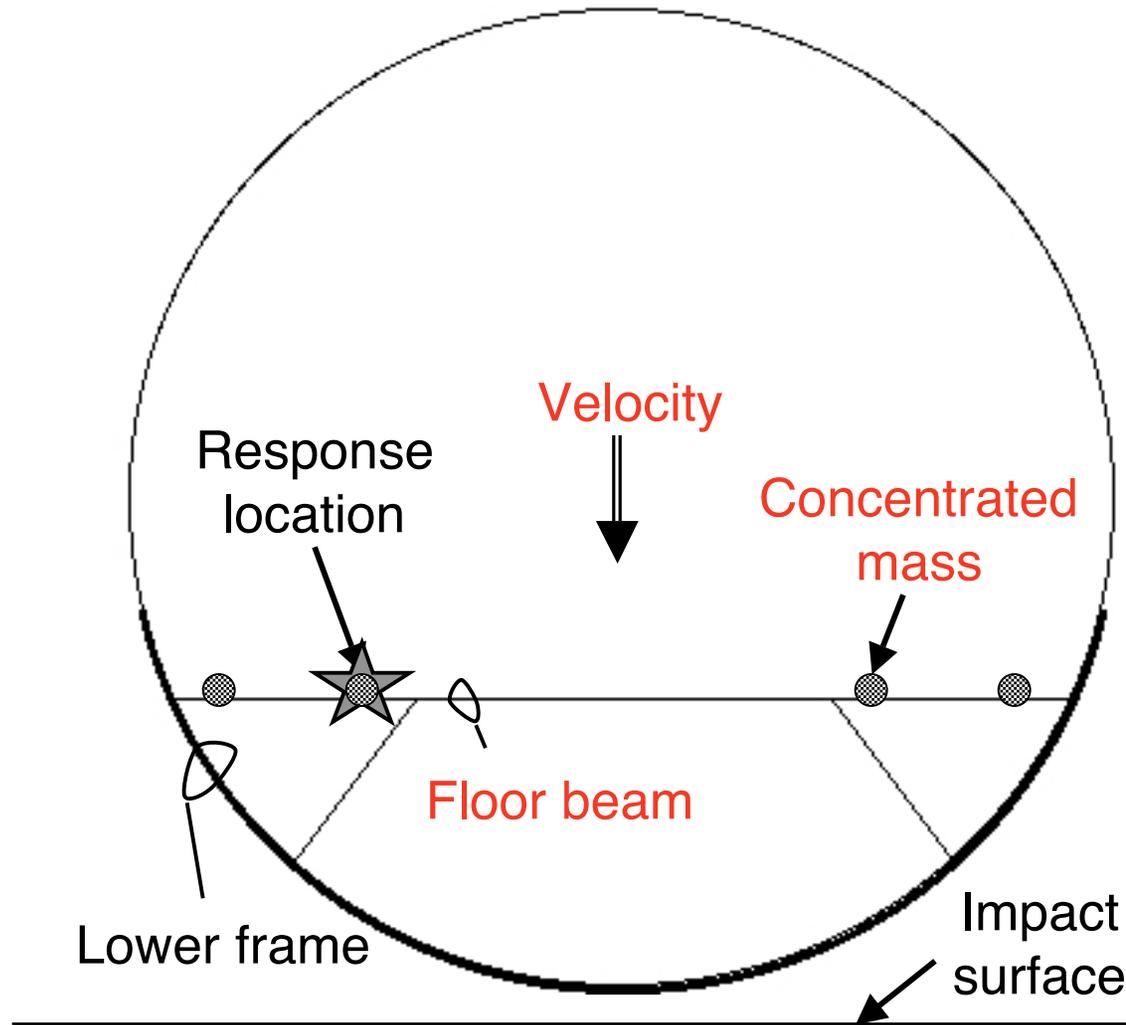
← before

during



Notice floor beam and accelerometers

# Schematic of Impact Dynamics Problem

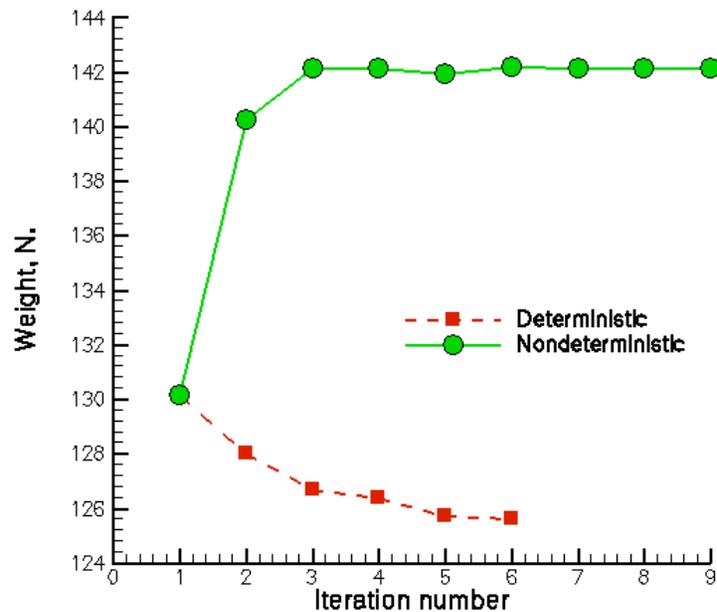


Note: Velocity and mass have normal distribution; floor beam has a maximum displacement

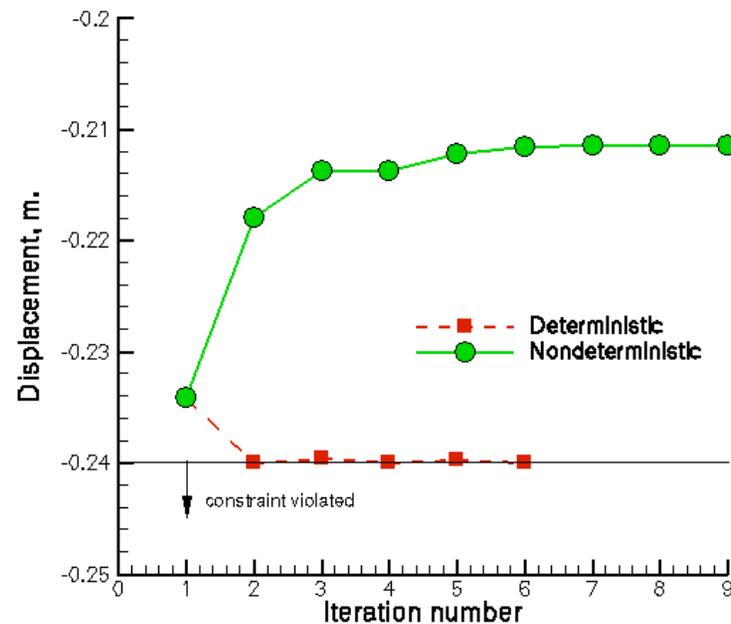
# Design for Crashworthiness

## A first attempt

Minimize: vehicle weight  
Subject to: max acceleration < survivable  
probability (displacement < allowable) > 75%



Weight increases



Displacement physically possible

# Lessons Learned

- Probabilistic constraints do steer optimizer away from troublesome regions of design space
- But, optimization under uncertainty is more difficult (e.g. more nonlinear problem) and expensive
- Approximate models (e.g. kriging) do reduce computational cost and provide credible solutions
- But, must rebuild approximate models as optimizer moves through design space

# Enabling Technologies

## In priority order

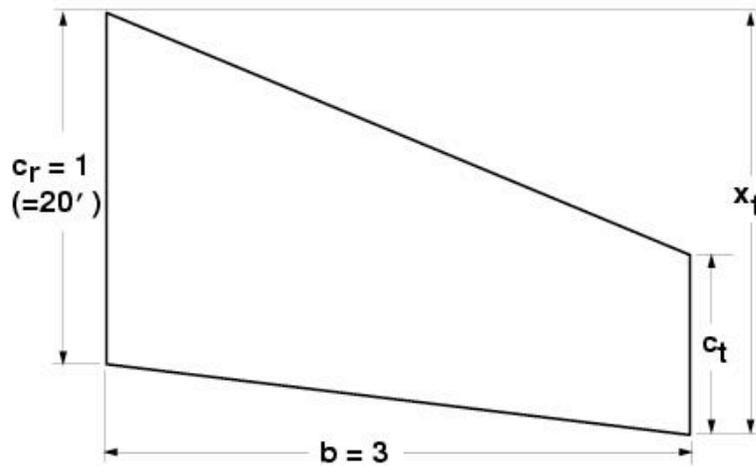
1. **Approximate engineering analysis for UQ**
2. Uncover designs with higher probability of success
3. Validate UBM for coupled multidisciplinary problems
4. Include physics-based UQ in conceptual design
5. Exploit variable-fidelity models to reduce expense
6. **Enable UBM for time dependent analysis**

# Outline

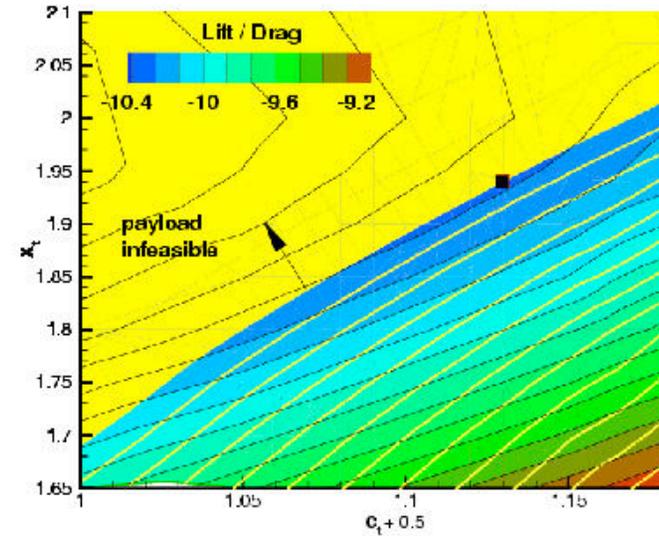
1. Three examples of optimization under uncertainty
  - Impact Dynamics
  - Coupled Aerodynamics/Structures
    - Gumbert, Newman, Hou, “Effect of Random Geometric Uncertainty on the Computational Design of a 3-D Flexible Wing”, AIAA-2002-2806.
  - Airfoil Shape Optimization
2. Lessons Learned
3. Enabling Technologies Identified

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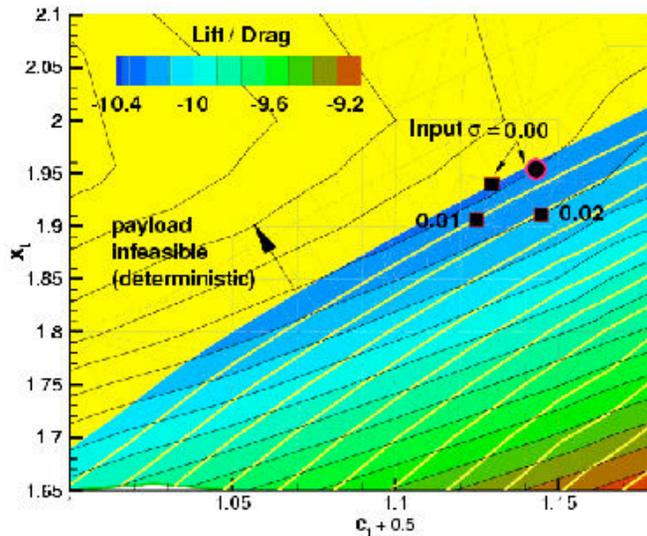
# Robust Shape Optimization for Flexible Wing with Planform Uncertainty



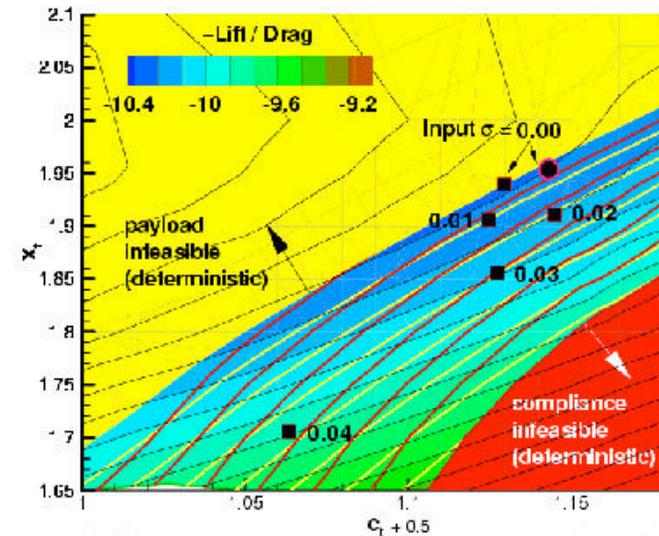
Wing planform variables  $c_t$  and  $x_t$ .



Conventional optimization result overlaid on level sets of deterministic objective and active constraint functions.



Probabilistic  $\sigma = 0.01$  &  $0.02$  robust optimization results overlaid on deterministic level sets.



Probabilistic  $\sigma = 0.03$  &  $0.04$  robust optimization results overlaid on deterministic level sets.



# Lessons Learned

- Adding uncertainty shrinks the design space
- Existence of deterministic solution does not guarantee a existence of non-deterministic solution
- Statistical First-order Second Moment Method was appropriate for this coupled multidisciplinary analysis

# Enabling Technologies

1. Approximate engineering analysis for UQ
2. Uncover designs with highest probability of success
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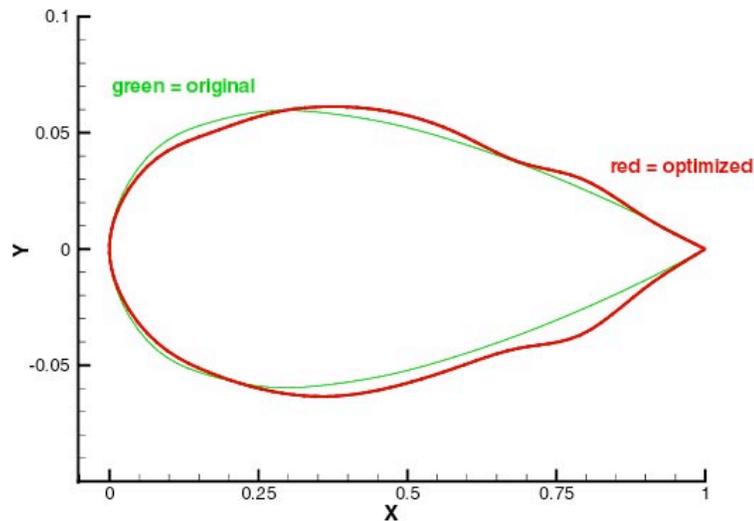
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  - Impact Dynamics
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  - Airfoil Shape Optimization
    - Li and Padula, "Performance Trades Study for Robust Airfoil Shape Optimization", AIAA-2003-3790.
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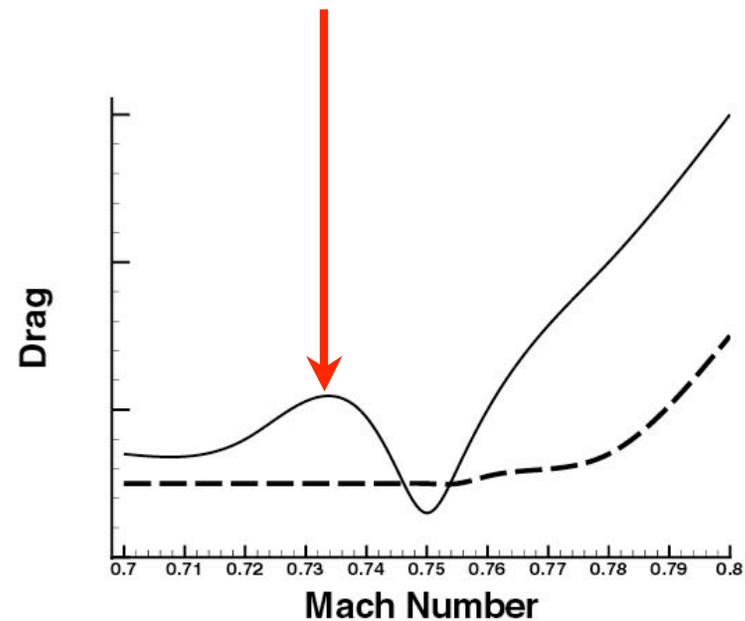
# Two Known Problems Associated With Lift Constrained Drag Minimization

$$\min_{D, \alpha_1, \dots, \alpha_r} \sum_{i=1}^r w_i c_d(D, M_i, \alpha_i) \text{ st } c_l(D, M_i, \alpha_i) \geq c_l^* \text{ for } 1 \leq i \leq r.$$

Noisy optimal airfoil

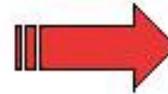


Off-design performance degradation

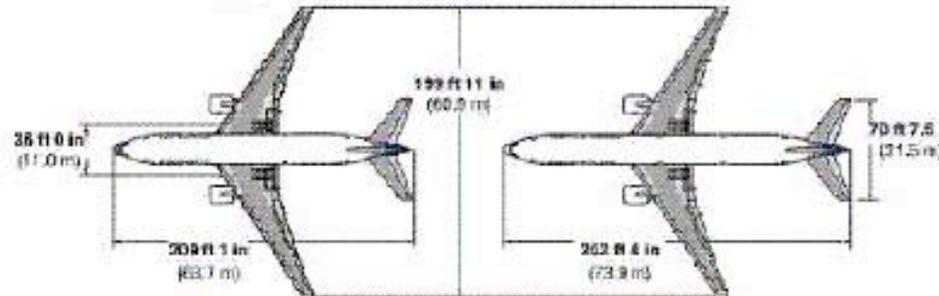


# Airfoil Shape Optimization for Design Teams

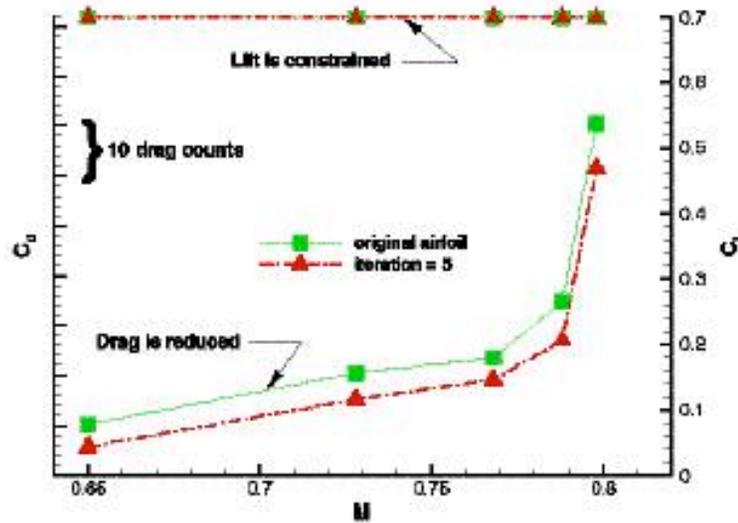
The baseline airplane with original airfoil



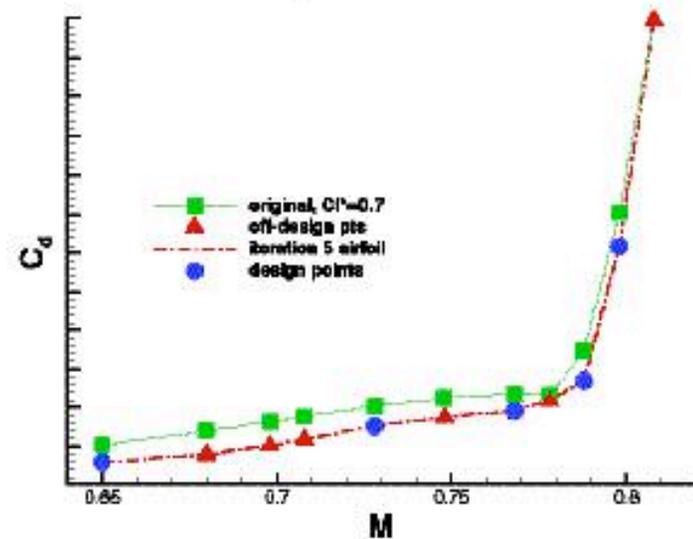
A bigger and faster airplane with optimized airfoil



Drag reduction at five specified design conditions



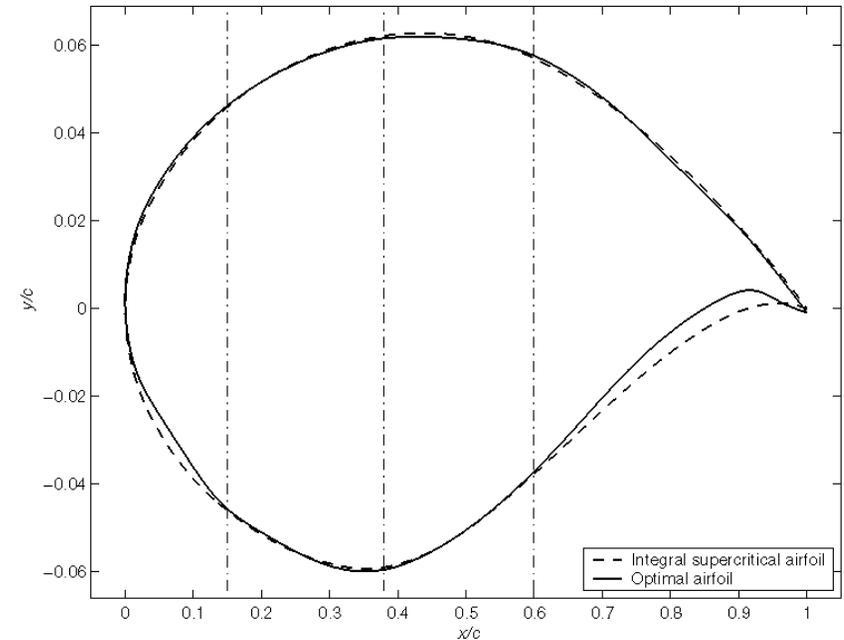
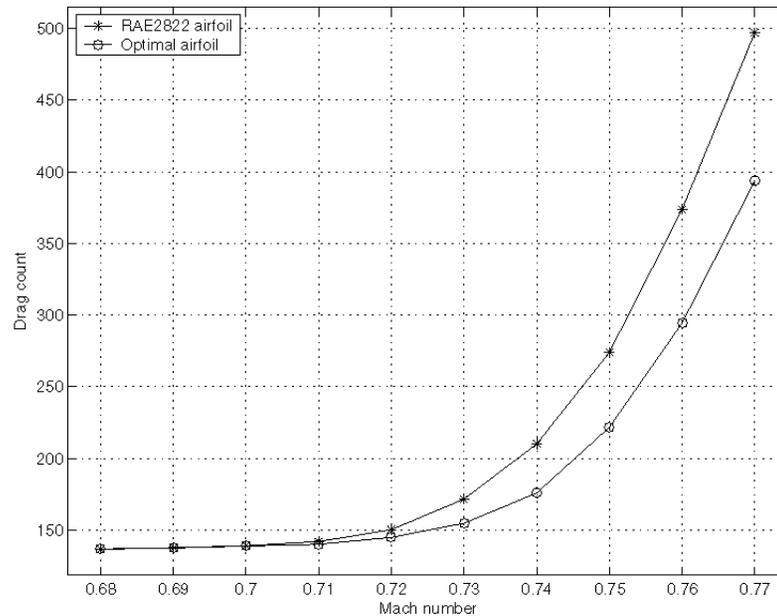
Post-optimization analysis at off-design conditions



# Airfoil Shape Optimization with uncertain operating conditions

4 RANS CFD solutions per iteration

Baseline & optimal airfoils



- What Has Been Accomplished?
  - Robust optimization directly minimizes wave drag for  $0.7 < \text{Mach } \# < 0.8$
  - New airfoil is similar to baseline but has less drag and no off-design performance hit
  - Use a small number of function evaluations and a large number of design variables

# Lessons Learned

- Relatively smooth optimum airfoils found even with 50-100 B-spline coefficients as design variables
- Success of optimization method depends on accurate gradient calculations (not available for all codes)
- Airfoils optimized including uncertainty are more acceptable to designers!!
- This work needs to be extended (e.g. 3-D wings)

# Enabling Technologies

1. Approximate engineering analysis for UQ
2. Uncover designs with highest probability of success
3. Validate UBM for coupled multidisciplinary problems
4. Include physics-based UQ in early design stages
5. Exploit variable-fidelity models to reduce expense
6. Enable UBM for time dependent analysis

# Concluding Remarks

- Optimization under uncertainty is a new concept for aerospace engineers at NASA Langley
- Impact dynamics and 3-D wing represent learning experiences for MDO branch
- Airfoil shape optimization pushes the state-of-the-art
- These studies uncover enabling technologies which require future investment