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Reliability Assessment of a Robust Design Under Uncertainty for a 3-D Flexible Wing

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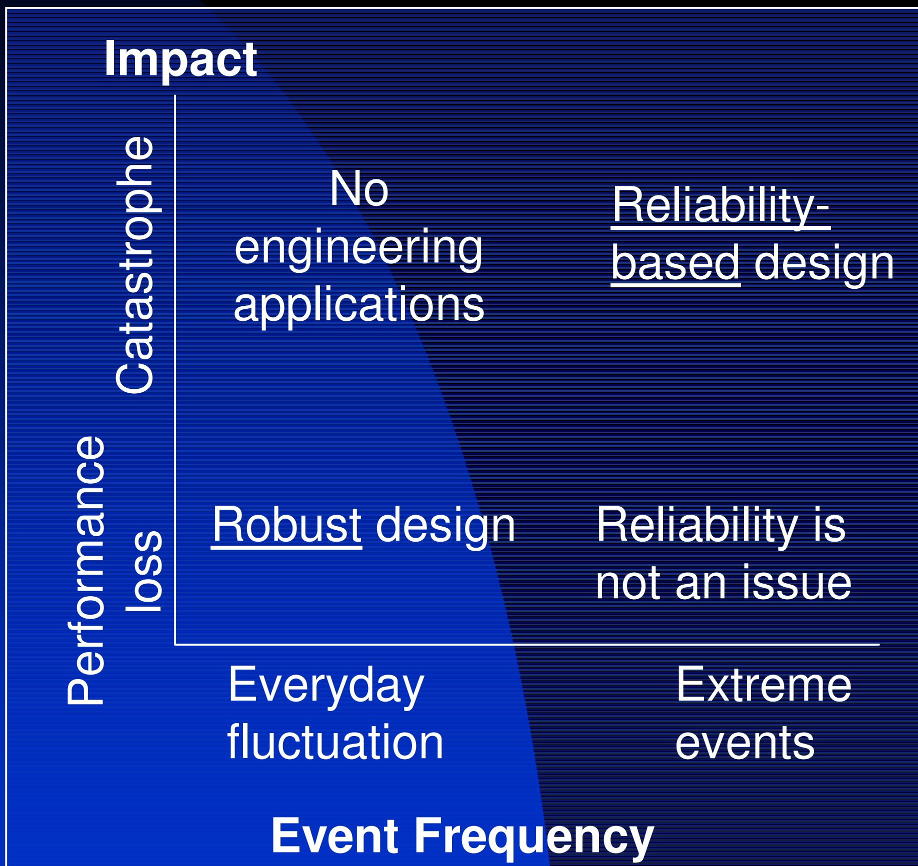
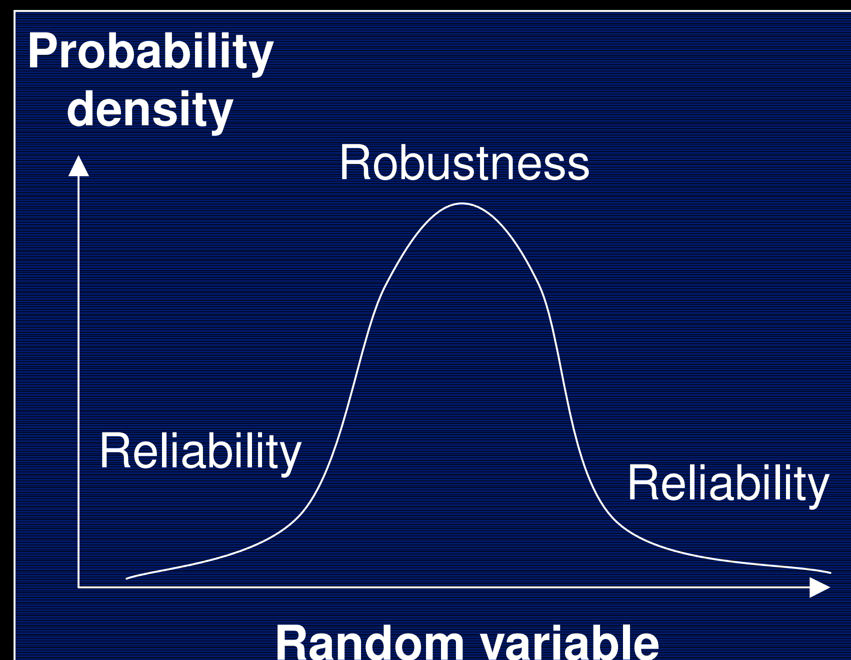
Uncertainty in Design





Outline

- Robust Optimization method
- Reliability Analysis method
- Static aeroelastic wing optimization problem
- Reliability assessment of design points from Robust Optimization
- Assess suitability of Reliability Analysis for optimization

First-Order Second Moment Approximation

- Random variables
  - statistically independent
  - normally distributed
  - with means $\bar{b} = \{\bar{b}_1, \cdots, \bar{b}_n\}$
  - and standard deviations $\sigma = \{\sigma_1, \cdots, \sigma_n\}$
- Expected values are

$$
\begin{aligned}
\bar{F} &= F(\bar{b}) \\
\sigma_F^2 &= \sum_{i=1}^{n} \left(\frac{\partial F}{\partial b_i} \sigma_i \right)^2
\end{aligned}
$$

Robust Optimization

- Stochastic design variables
- Objective function cast as expected value of output function
- Constraints cast as a probability of satisfaction greater than $\Phi(k)$

$$\min_{\bar{b}} \quad \Psi(\bar{F}, \sigma_F; \bar{Q}, \bar{b})$$

$$\text{subject to} \quad g(\bar{F}; \bar{Q}, \bar{b}) + k\sigma_g \leq 0$$

- 2nd derivatives for gradient-based optimization

Reliability Analysis (FORM)

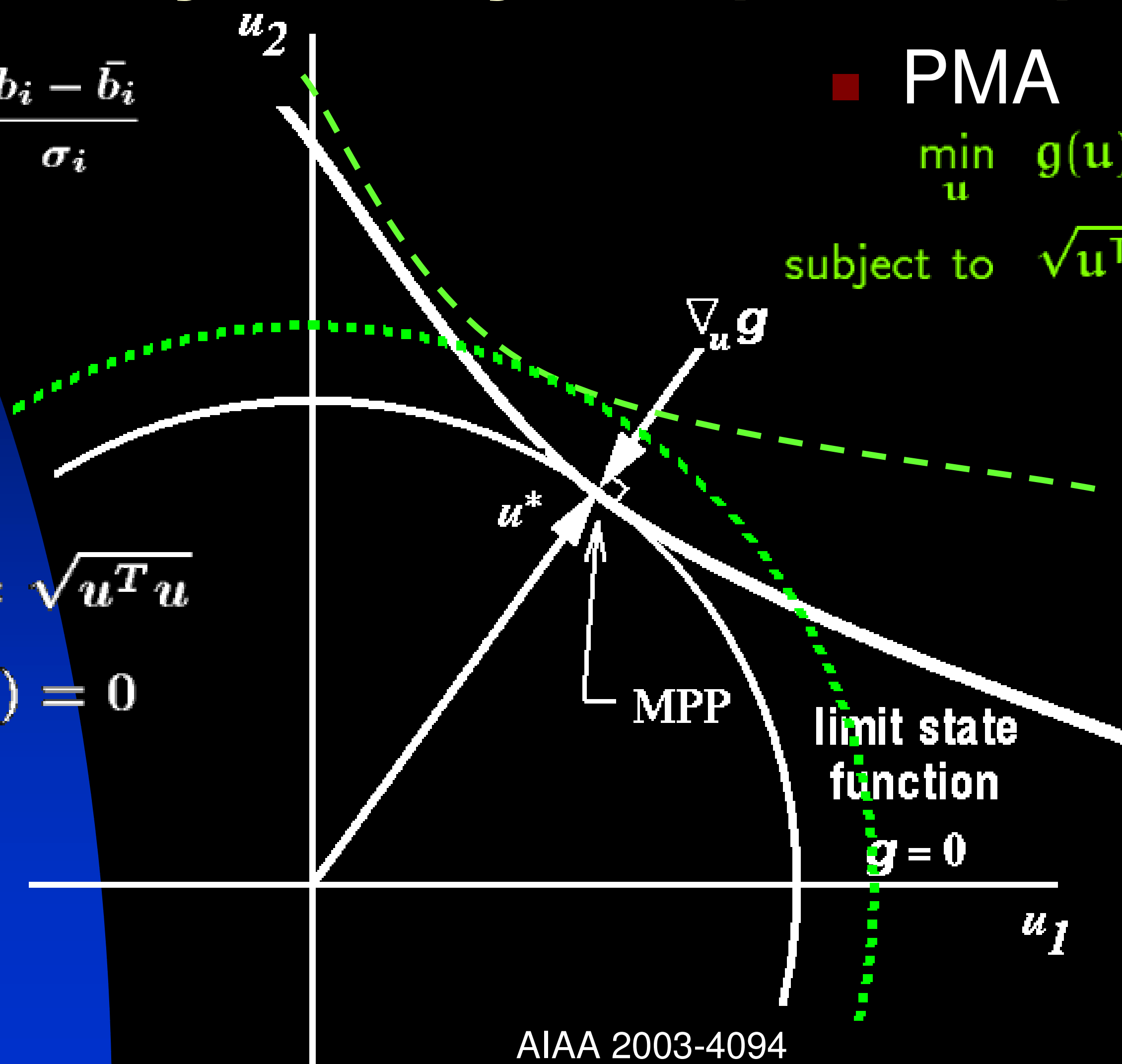
$$u_i = \frac{b_i - \bar{b_i}}{\sigma_i}$$

- PMA

$$\min_{u} \quad g(u)$$
$$\text{subject to} \quad \sqrt{u^T u} - \beta_0 = 0$$

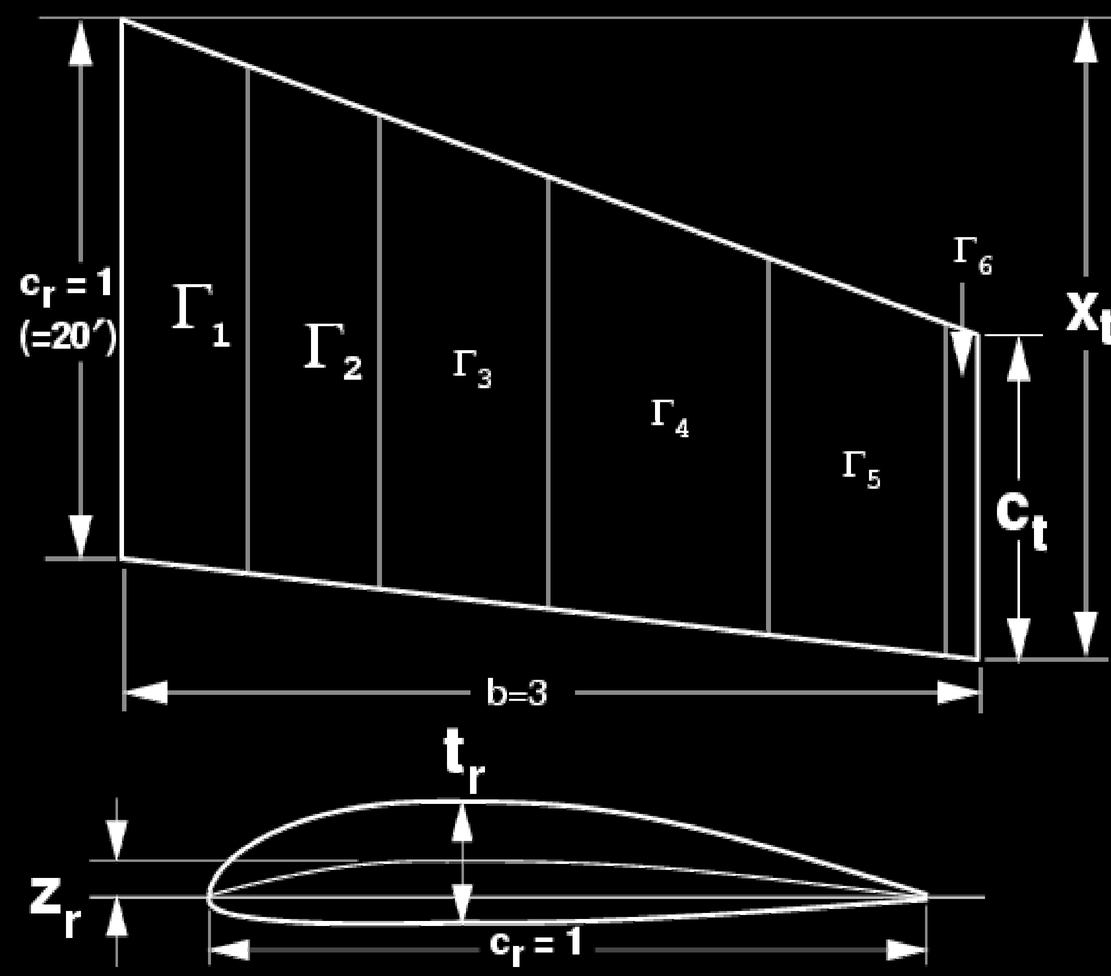
- RIA

$$\min_{u} \quad \beta = \sqrt{u^T u}$$
$$\text{subject to} \quad g(u) = 0$$



3-D Flexible Wing

- Two DV
  - c_t , x_t
- Four DV
  - Γ_1, Γ_2
  - z_r , t_r



Objective and Constraints

- Objective function: $-(L/D)^2$
- Constraints:
  - Minimum payload $L - W$

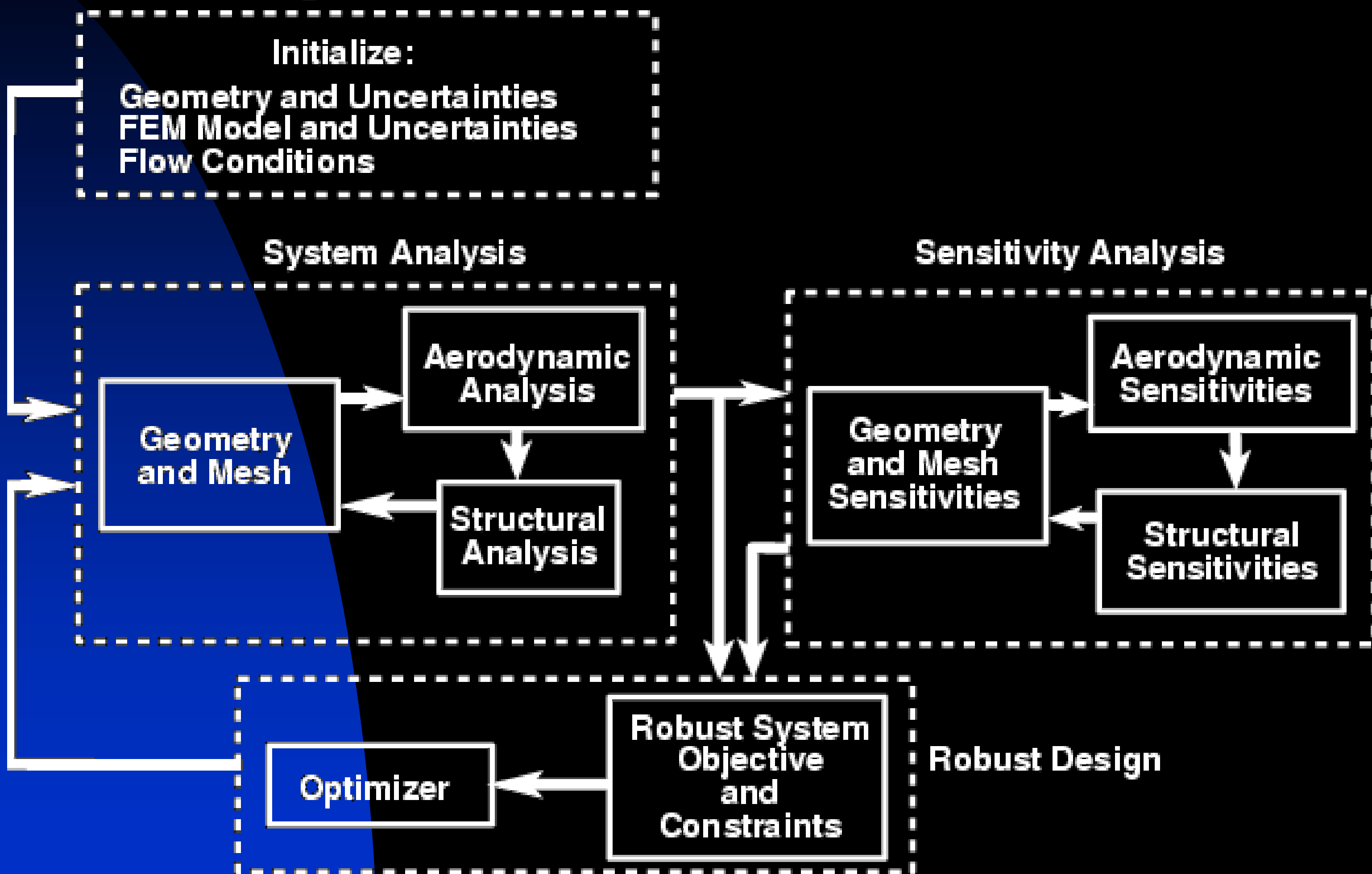
  where $L = C_L S q_\infty$

  - Maximum compliance $V = \oint p u \cdot \hat{n} ds$
  - Maximum pitching moment C_m
  - Minimum LE radius
- Solution of coupled Euler and FEM analysis, R(Q,u,b)=0, $M_\infty = 0.8, \alpha = 1°$

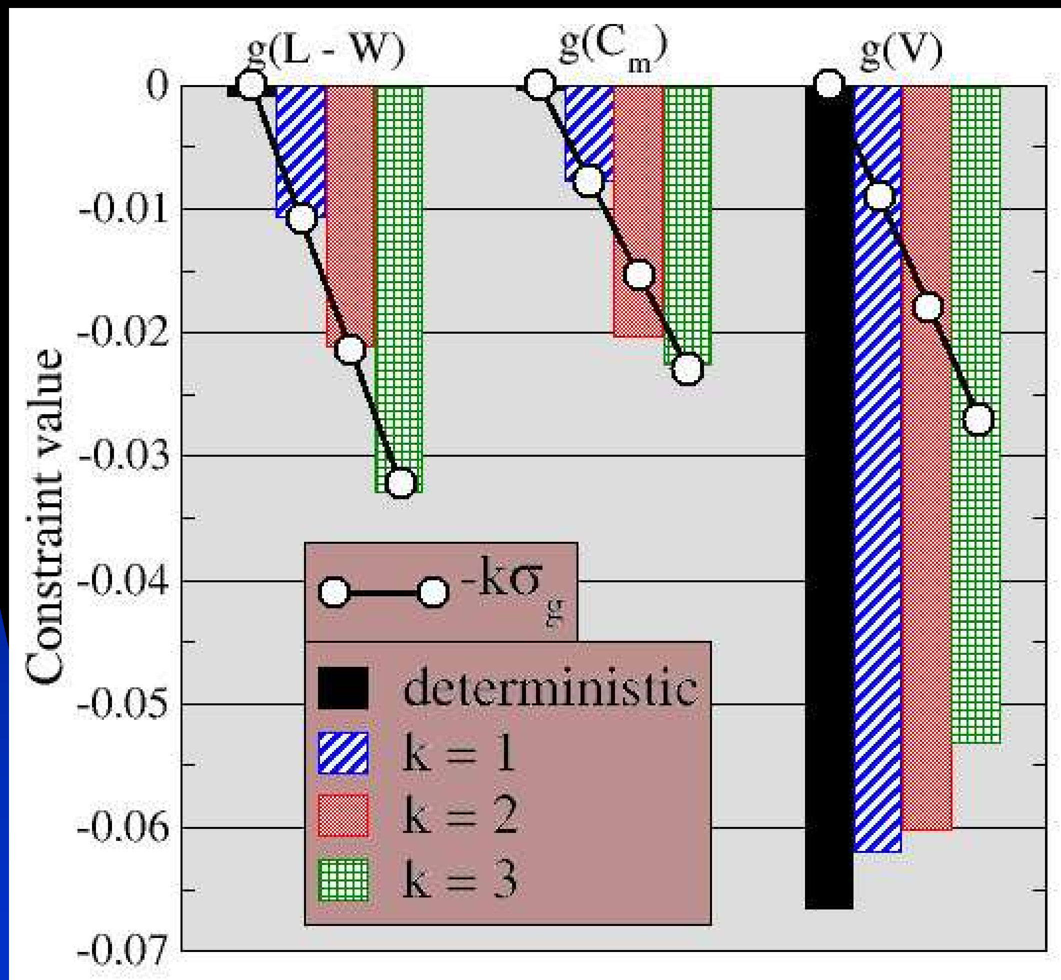
Coupled CFD/FEM MDA

- Parameterized surface geometry
- Surface mesh deformation
- Volume mesh deformation
- 3-D Euler CFD
- Linear FEM: skin, spars, ribs
- Sensitivity analyses also coupled

Optimization Process



Robust Results – 4DV



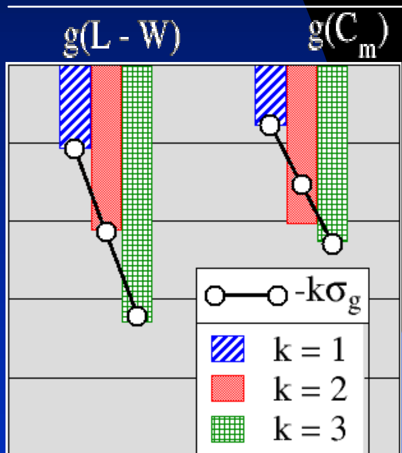
Merit Functions for Reliability Assessment

Robust Design

$g = \bar{g} + k\sigma_g \le 0$

Reliability Analysis

$P(g_p) = \Phi(-\beta)$



k

RIA: $g = 0 \Rightarrow \beta^*$

$\bar{g} + k\sigma_g$

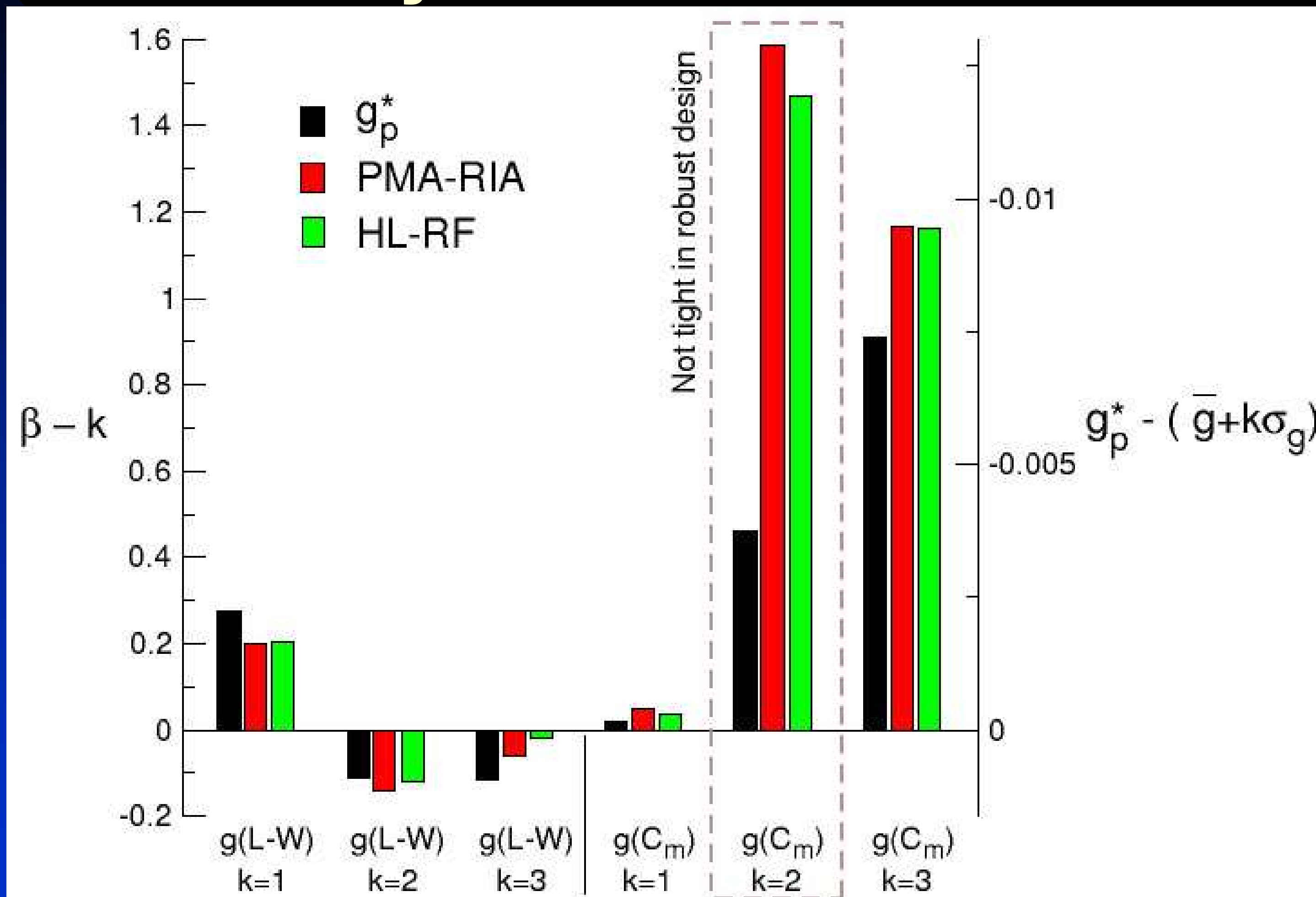
PMA: $\beta_0 \Rightarrow g_p^*$

Assessment functions:
(near zero is good)

$\beta^* - k$

$g_p^* - (\bar{g} + k\sigma_g)$

Reliability Assessment – 4DV



Impact of Reliability Analysis on Optimization (RBDO)

- Inner optimization for RA
  - Required for each active constraint
  - Need screening process
- Sensitivity derivatives $\frac{\partial \beta}{\partial DV}$ & $\frac{\partial g^*}{\partial DV}$ required for gradient-based optimization
  - First derivative from inner optimization
  - Second derivative not required
- Accuracy and cost

RA Sensitivity Derivatives – 4DV

DV	Constraint	k	$\frac{\partial\beta}{\partial DV}$, HL-RF analytic	ratio	$\frac{\partial\beta}{\partial DV}$, PRIA analytic	ratio	$\frac{\partial g^*}{\partial DV}$, PMA analytic	ratio
Γ_1	$g(L - W)$	1	-6.530	1.042	-6.530	1.004	.06952	0.992
Γ_1		2	-6.528	0.940	-6.527	1.008	.06917	0.998
Γ_1		3	-6.512	1.072	-6.511	1.011	.06881	0.998
Γ_1	$g(C_m)$	1	-9.457	0.942			.05650	1.026
Γ_1		2	-9.778	†	-9.778	1.000	.05799	0.941
Γ_1		3	-9.984	†	-9.984		.05949	1.036
t_r	$g(C_m)$	3	-471.5		-471.5		2.802	1.008
z_r		2	-818.4	†	-818.5	0.962	4.835	0.998
z_r		3	-816.3		-816.3	0.990	4.846	1.000
Γ_2		2	-22.47	1.099	-22.47	0.958	.1332	0.999
Γ_2		3	-23.02	1.018	-23.02	1.003	.1372	0.991

† nonconvergent RIA for at least one side of the FD

RA Cost Comparison

			HL-RF		PMA		PRIA	
nDV	Constraint	k	Nfunc	Nderiv	Nfunc	Nderiv	Nfunc	Nderiv
2	g(L-W)	1	3	3	3	3	6	5
2		2	3	3	3	3	6	5
2		3	3	3	3	3	6	5
2		4	5	5	3	3	10	8
2	g(V)	3	3	3	3	3	9	7
2		4			3	3	10	8
4	g(L-W)	1	3	3	3	3	6	5
4		3	3	3	3	3	6	5
4	$g(C_m)$	1	4	4	3	3	6	5
4		2	3	3	3	3	6	5
4		3	3	3	3	3	9	7

Summary & Conclusions

- Three reliability assessments more consistent with robust results at $k \leq 3$
- PMA-based RIA eliminates non-convergence problem of HL-RF, but more expensive
- Reliability analysis derivatives suitable for RBDO
- Robust design requires second derivatives; reliability analysis requires additional optimization
- Use of PMA in design similar to Robust Design
- Use of PMA as screen for PRIA

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