# Effect of Geometric Uncertainty on the Computational Design of a 3-D Flexible Wing

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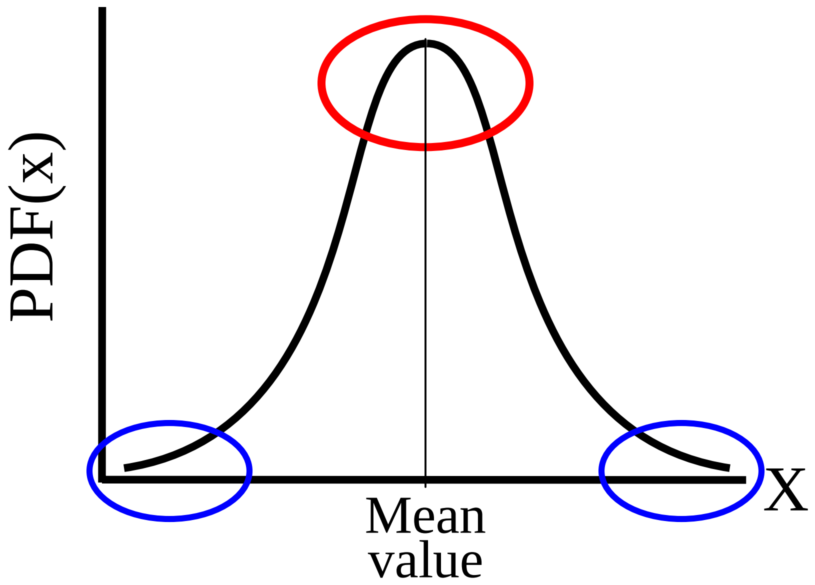
# Outline

- Uncertainty based design concepts
- Integrated Statistical Approach
- Application to 3-D flexible wing
- Sample results
  - 2 uncertain design variables
  - 4 uncertain design variables
- Concluding remarks

# Uncertainty-Based Design (UBD)

- **Reliable** or **robust**
  - **infrequent, catastrophic**
  - **frequent, degraded performance**
- Sources of uncertainty
  - **inherent**
  - **model**
  - **human factors**



# Integrated Statistical Approach

- Uncertainty identification and quantification
- Uncertainty propagation
- Robust design

# Uncertainty Propagation

First-Order Second Moment (FOSM) Method

Given: $\mathbf{b} = \{b_1, \ldots, b_n\}$

with means, $\bar{\mathbf{b}} = \{\bar{b}_1, \ldots, \bar{b}_n\}$

and standard deviations $\boldsymbol{\sigma} = \{\sigma_{b_1}, \ldots, \sigma_{b_n}\}$

First-order Taylor series:

$$F(\mathbf{b}) = F(\bar{\mathbf{b}}) + \sum_{i=1}^{n} \frac{\partial F}{\partial b_i}(b_i - \bar{b}_i)$$

then expected values are

$$\bar{F} = F(\bar{\mathbf{b}}) \quad \text{and} \quad \sigma^2_F = \sum_{i=1}^{n}\left(\frac{\partial F}{\partial b_i}\sigma_{b_i}\right)^2$$

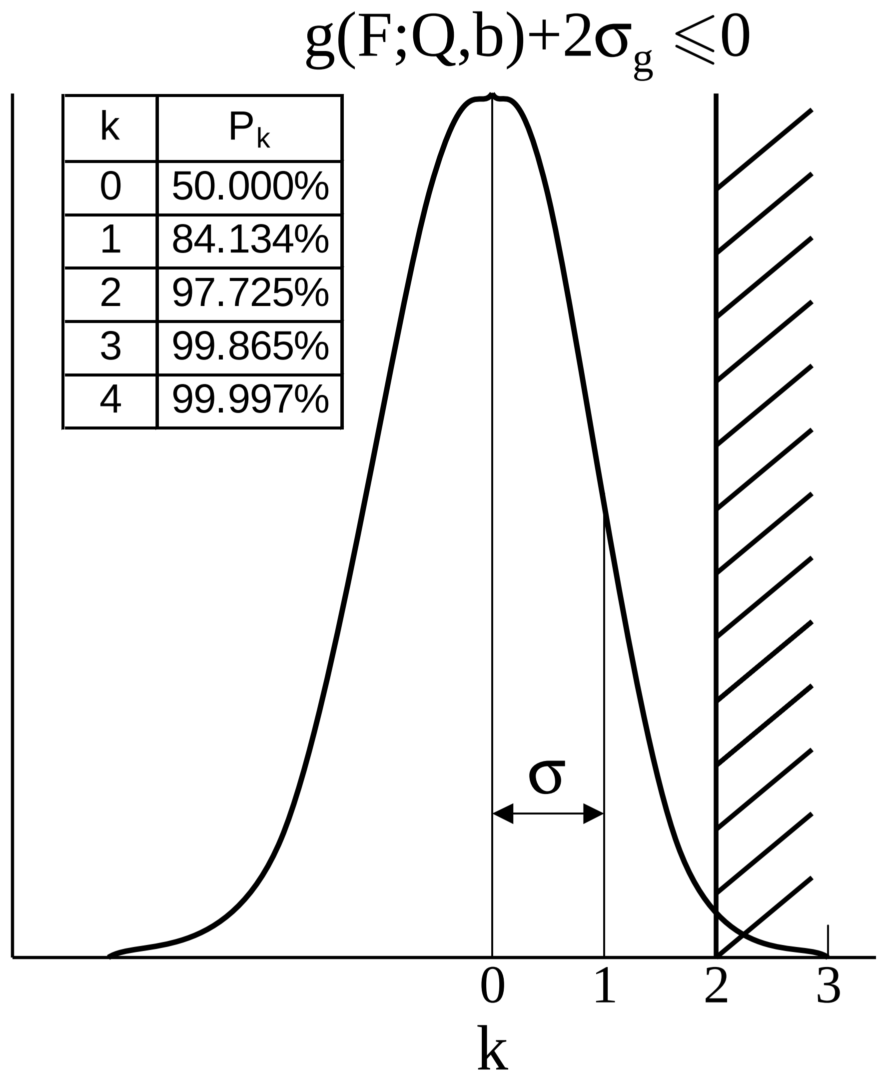
# Robust Design

$$\min \Psi = \Psi(\bar{F}, \sigma_F; \bar{\mathbf{Q}}, \bar{\mathbf{b}})$$

subject to

$$g(\bar{F}; \bar{\mathbf{Q}}, \bar{\mathbf{b}}) + k\sigma_g \leq 0$$

$k\sigma_g$ represents the desired safety factor for probabilistic constraint satisfaction

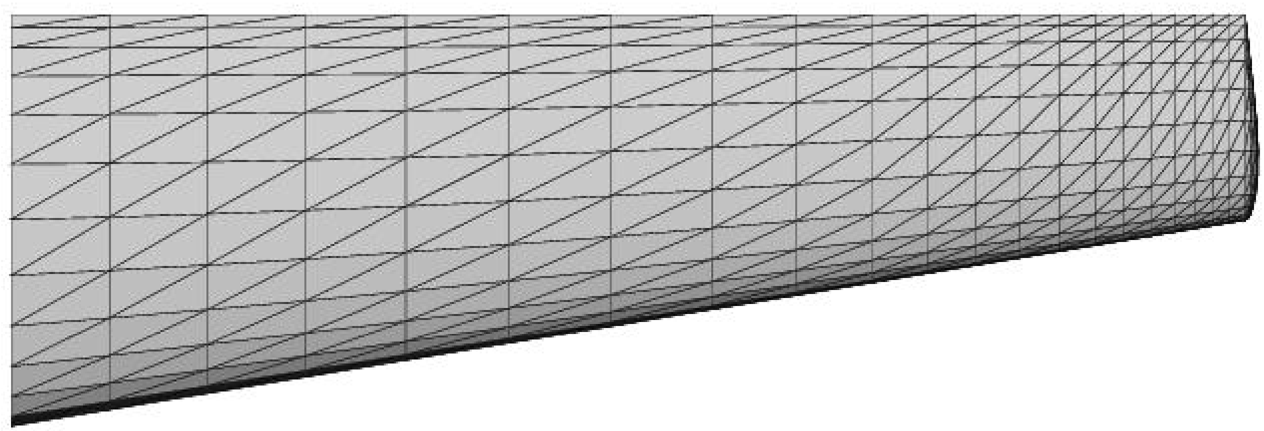



| k | $P_k$ |
|---|---|
| 0 | 50.000% |
| 1 | 84.134% |
| 2 | 97.725% |
| 3 | 99.865% |
| 4 | 99.997% |

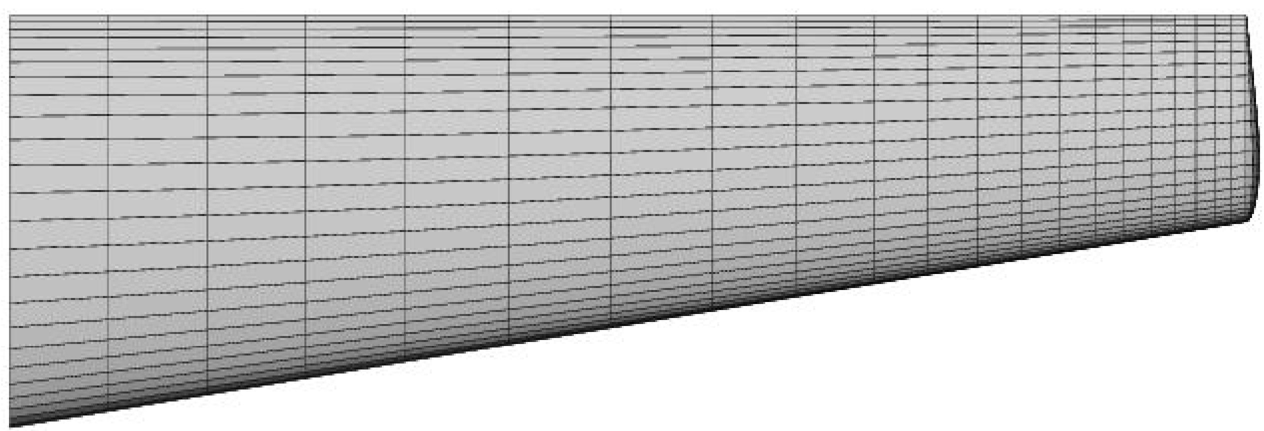
# Application to 3-D Flexible Wing

## Euler CFD and FE structural models

FEM mesh
3251 elements
583 nodes



Euler CFD mesh
C-O topology
73x25x25 volume
49x25 on wing



# Application to 3-D Flexible Wing

## Optimization problem statement

minimize: $-(L/D)^2$

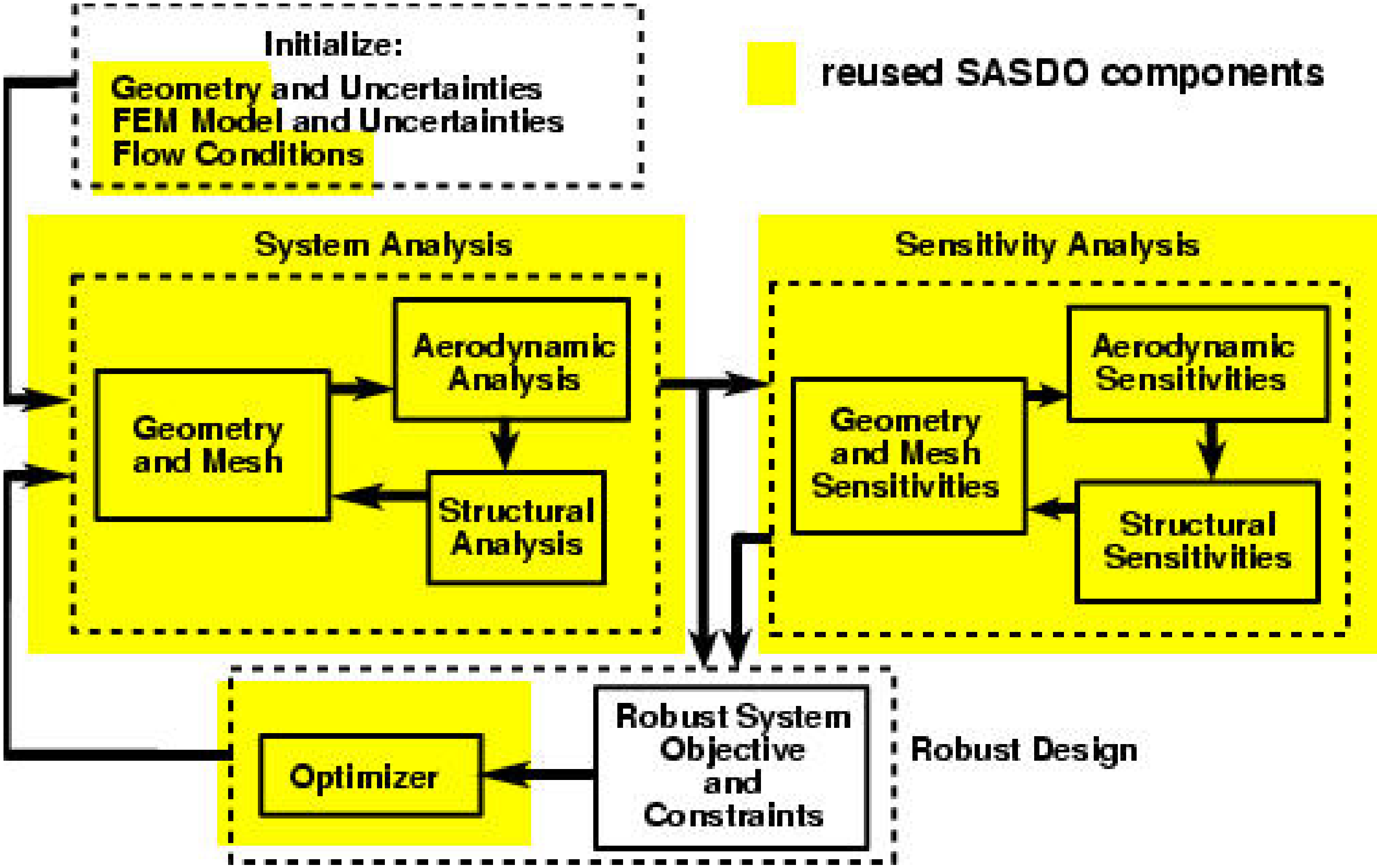
at $M_\infty=0.8$, $\alpha=1^o$

subject to constraints:

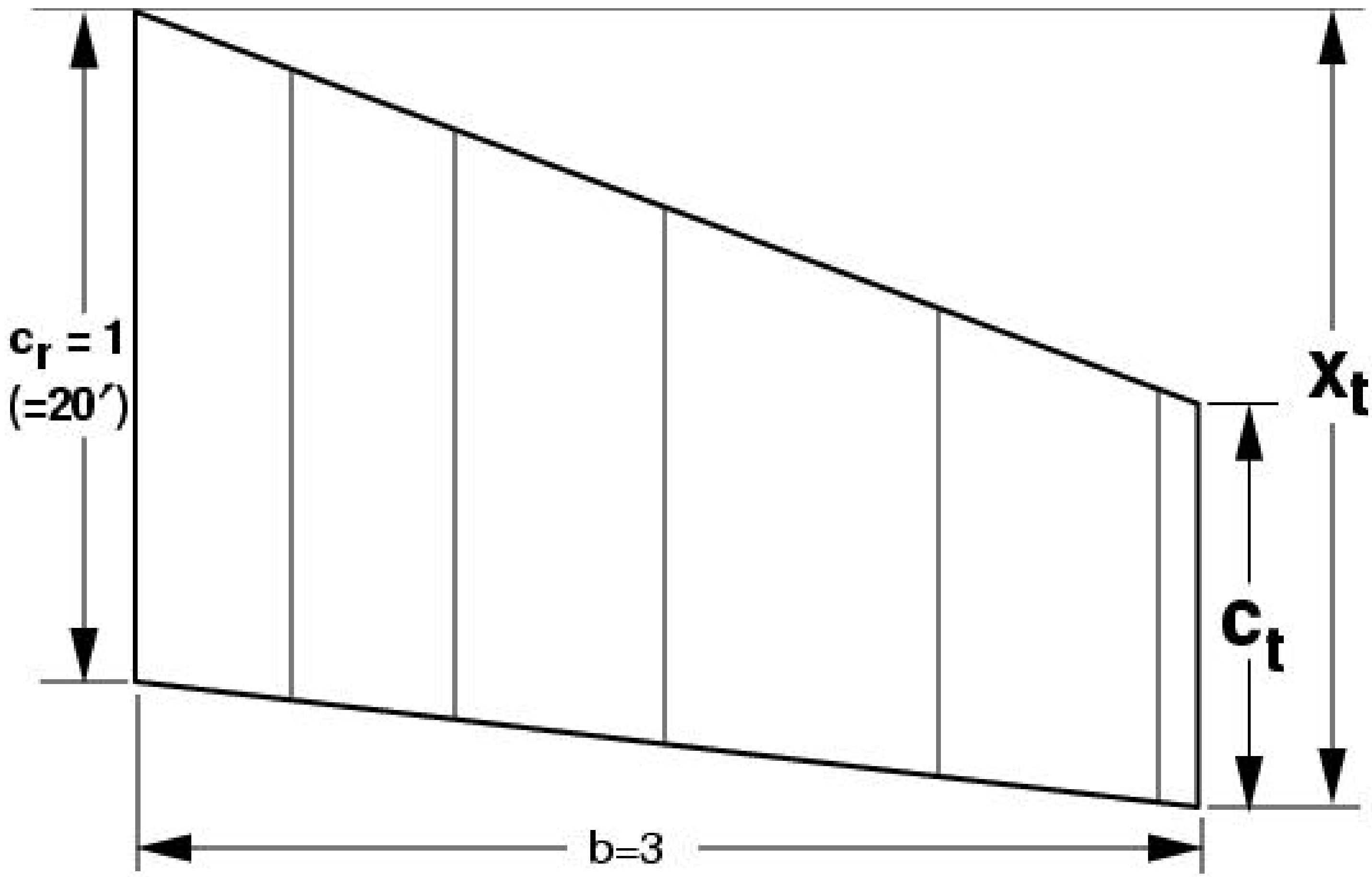
- $g_1 = L - W \geq$ minimum payload,
  where $L = C_L q_\infty S$
- $g_2 = V = \oiint p\mathbf{u} \cdot \hat{\mathbf{n}}\, ds \leq$ maximum compliance
- $g_3 = C_m \leq$ maximum pitching moment

# Application to 3-D Flexible Wing

## Schematic of robust design process

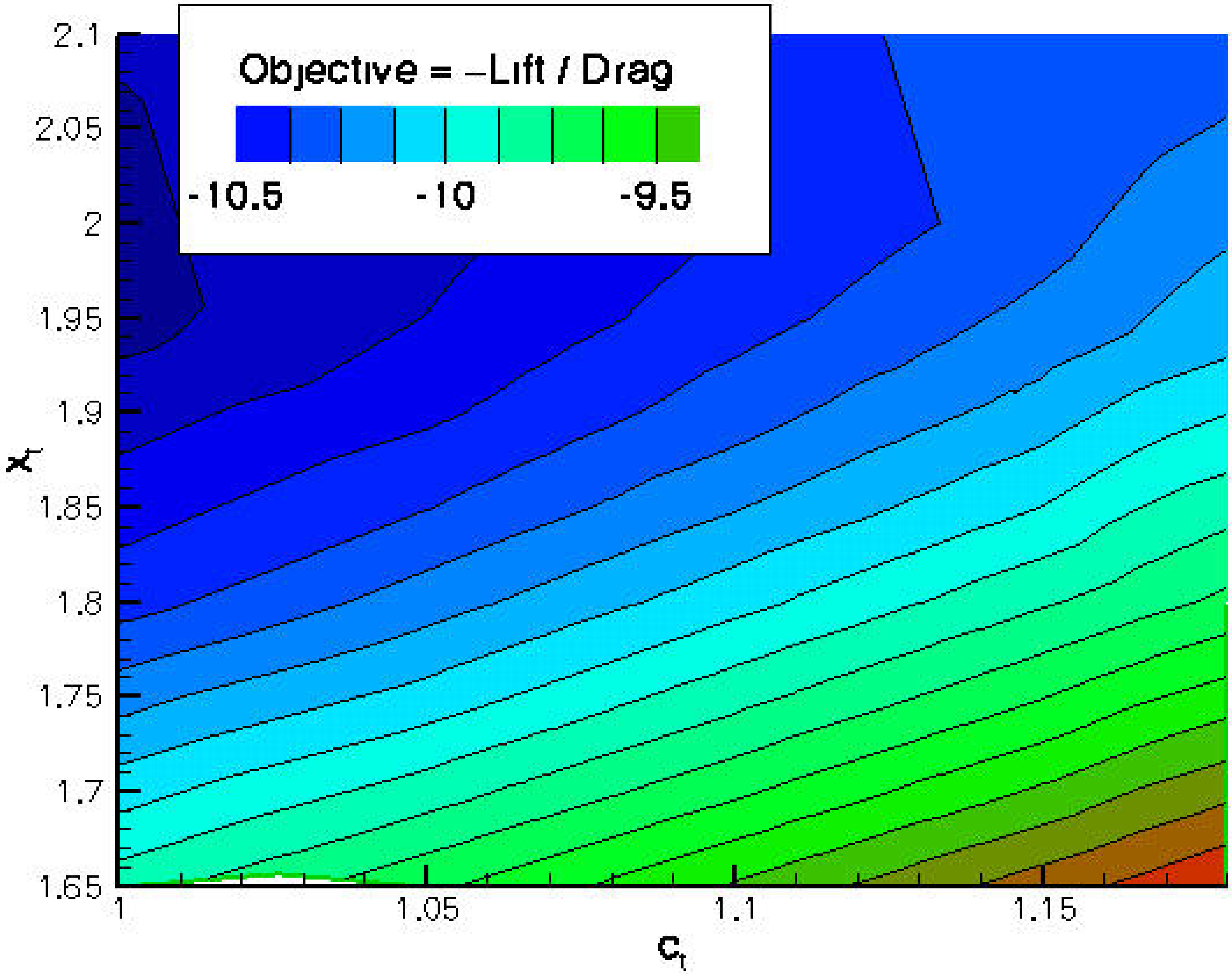


# Two-DV Problem



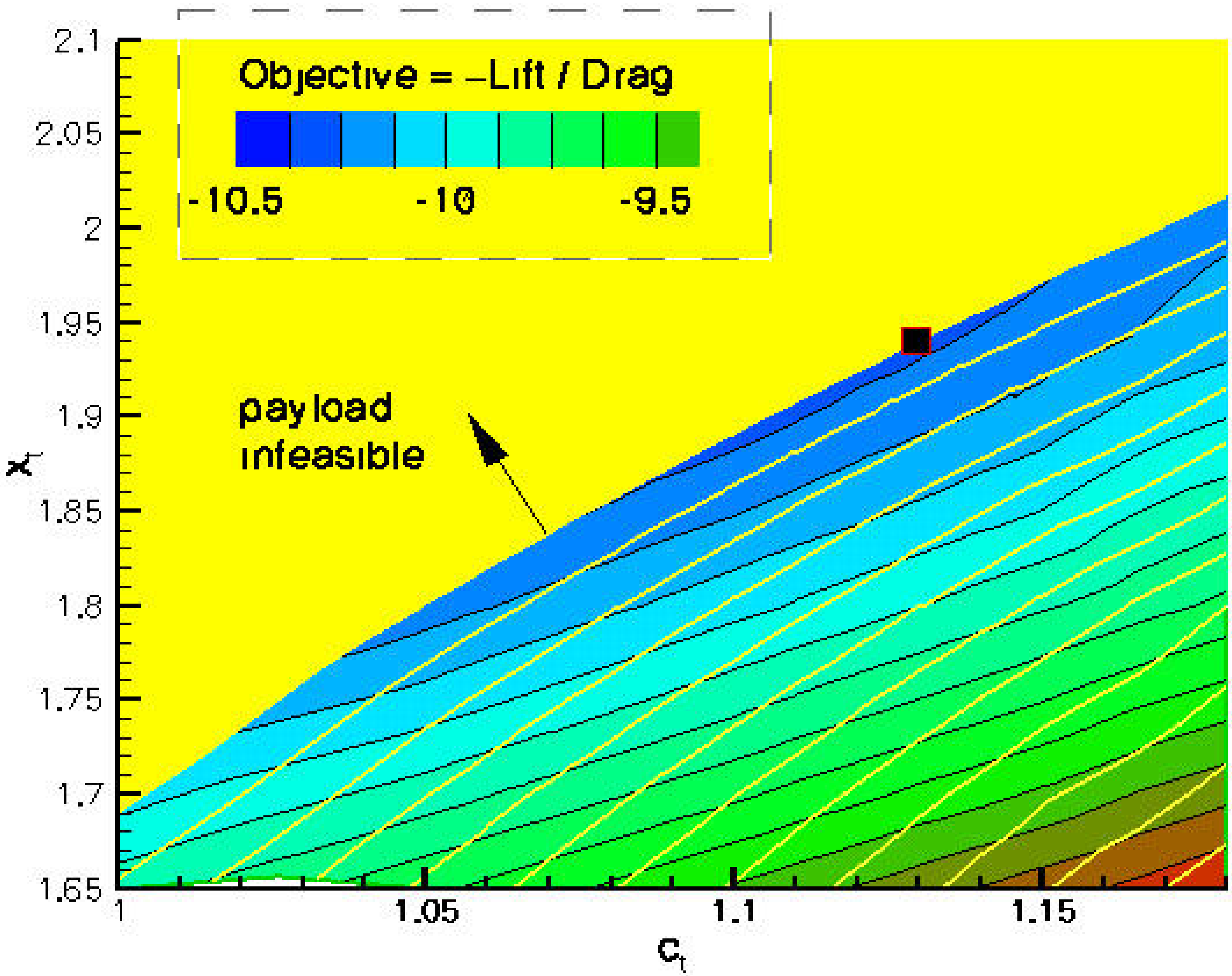
# Two-DV Sample Results

## Objective function contours



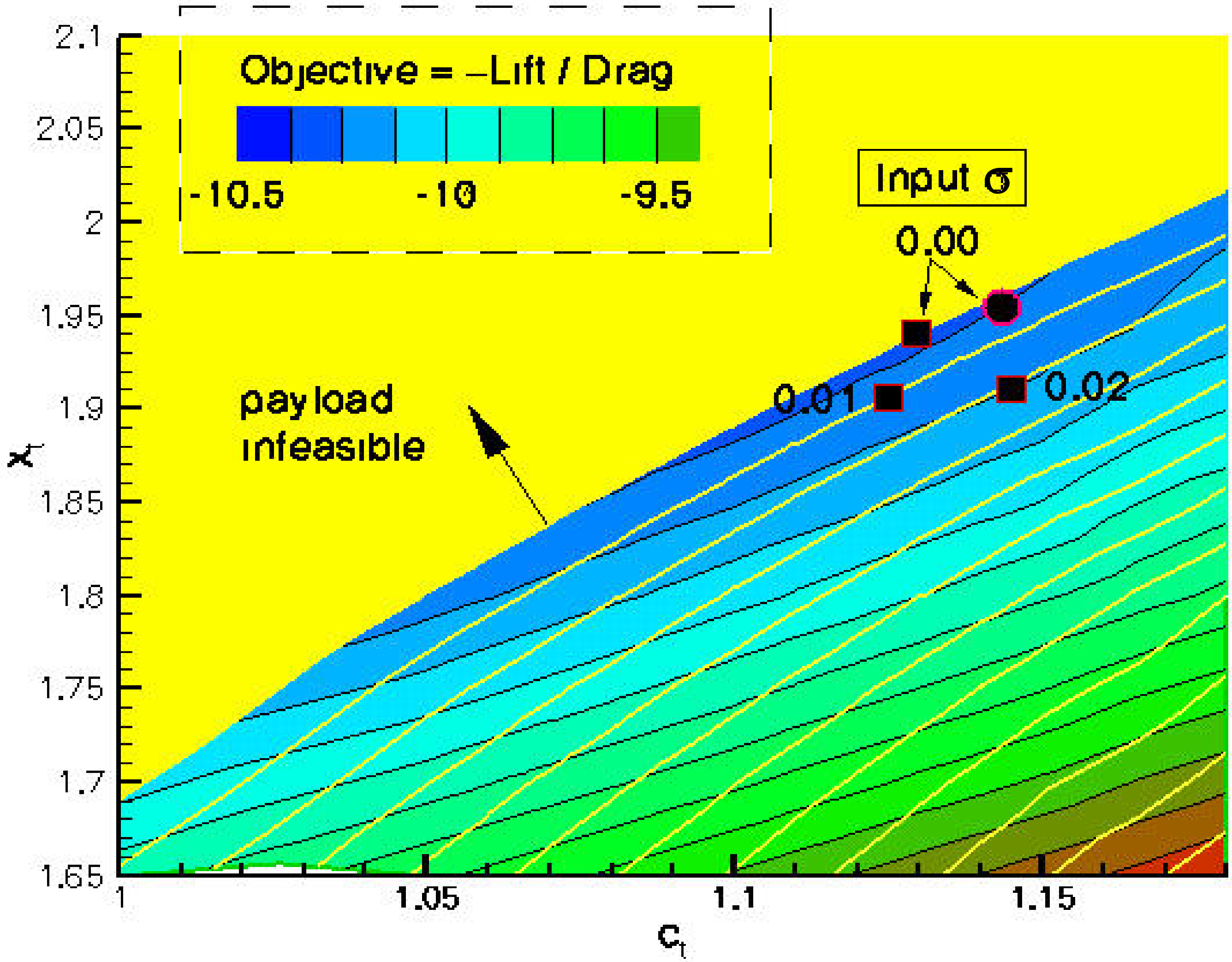
# Two-DV Sample Results

## Deterministic result



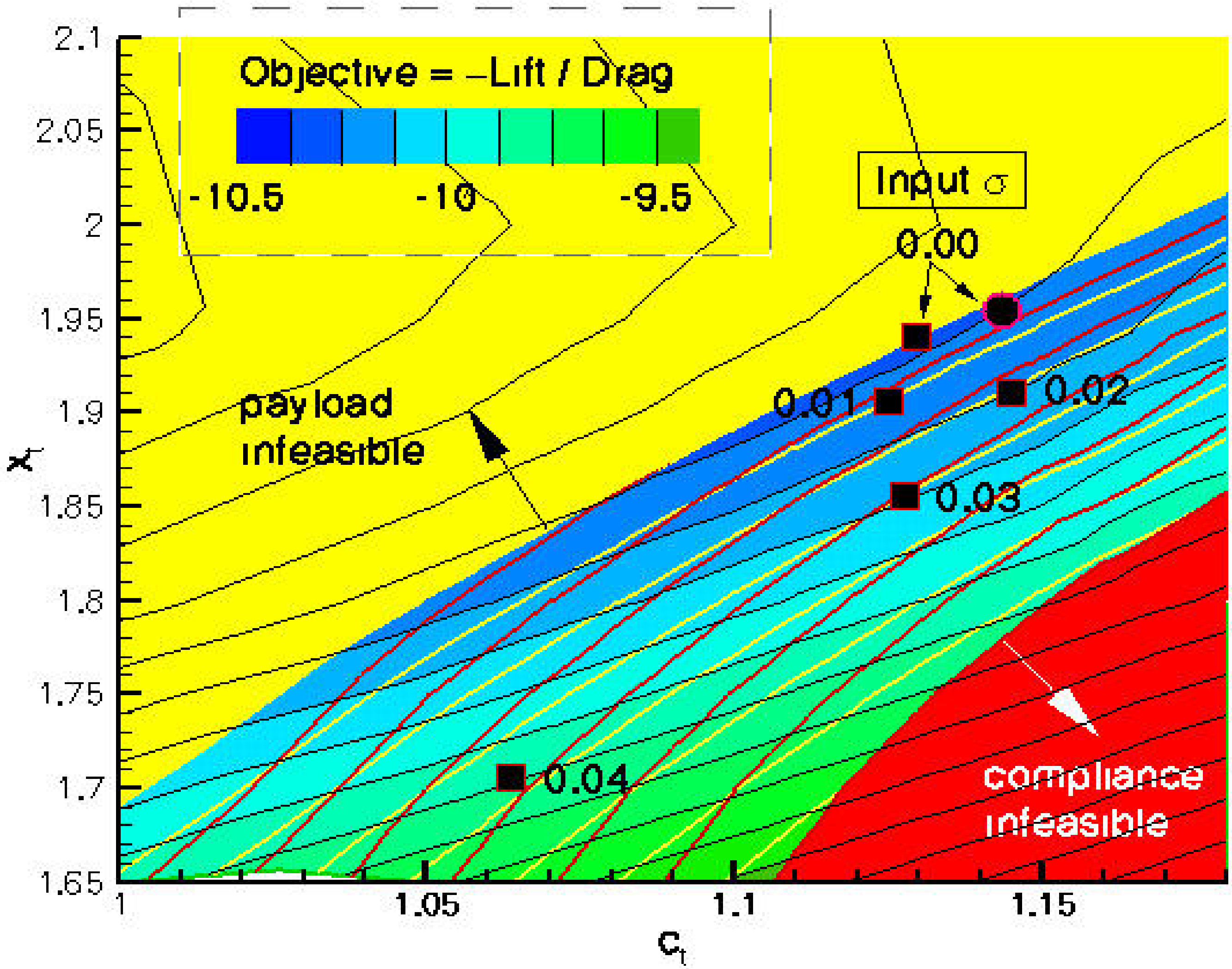
# Two-DV Sample Results

## With input uncertainty



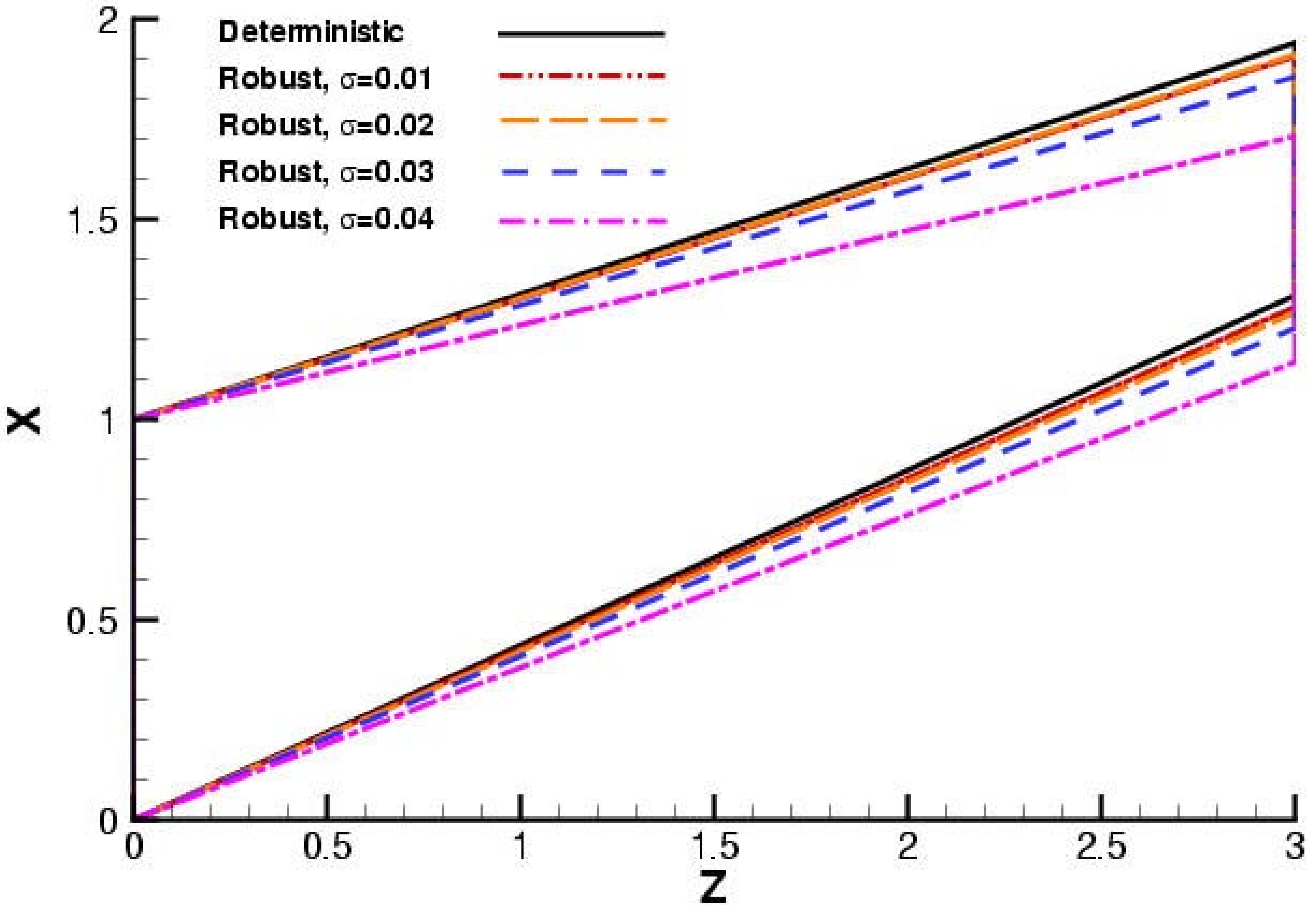
# Two-DV Sample Results

## With larger input uncertainty

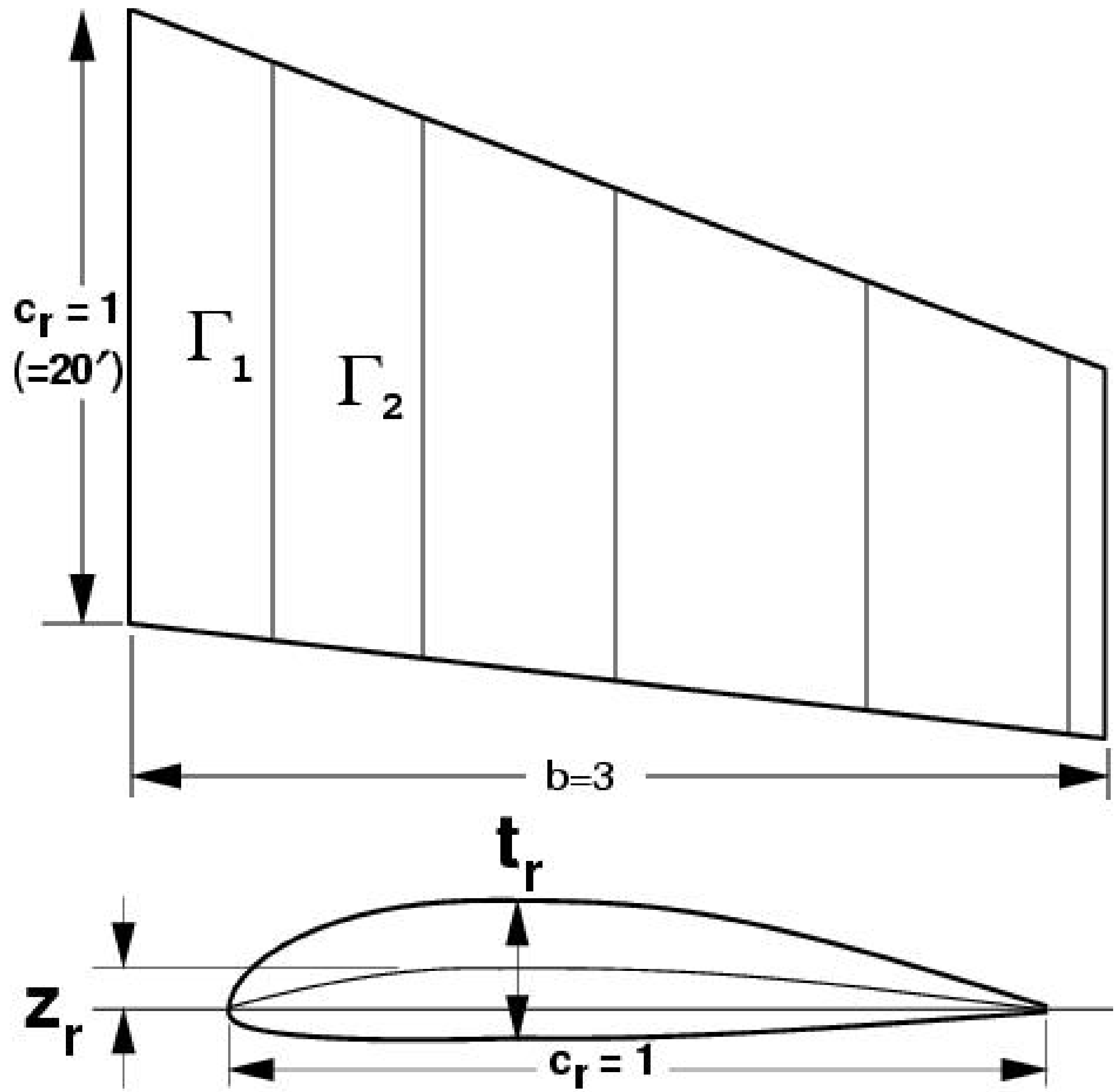


# Two-DV Sample Results

$$M_\infty = 0.8,\ \alpha = 1^o$$



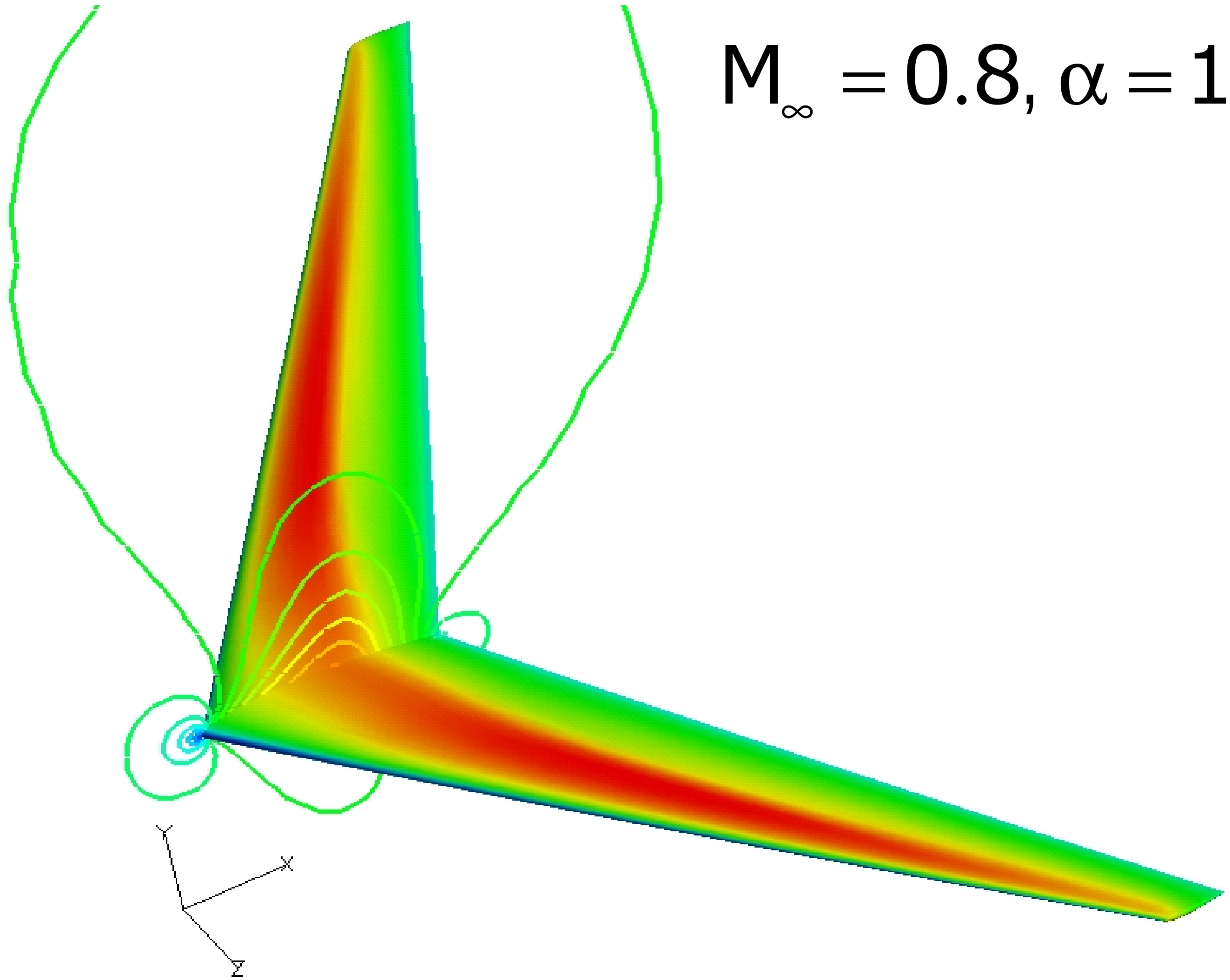
# Four-DV Problem



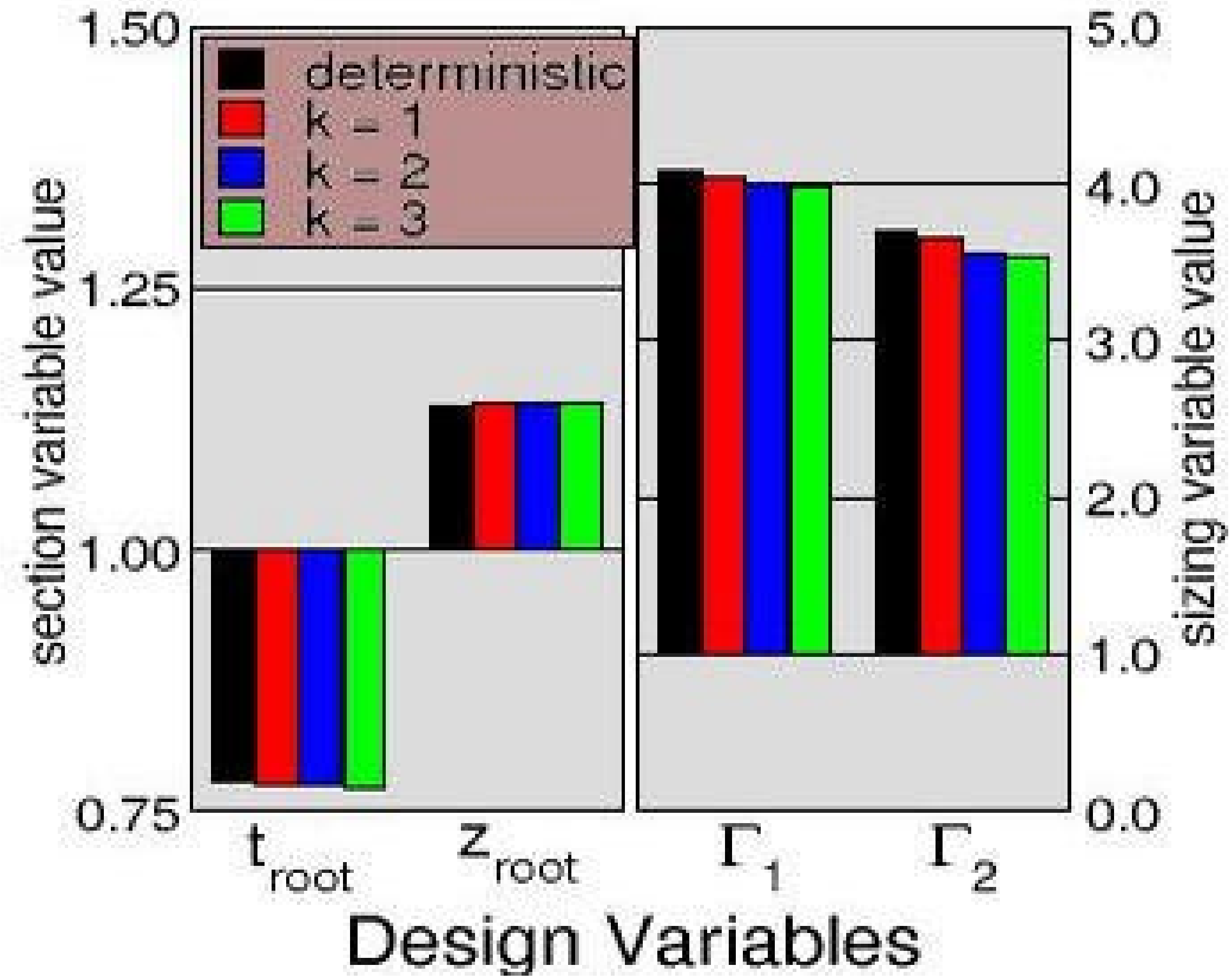
# Four-DV Sample Results

## Initial deterministic design

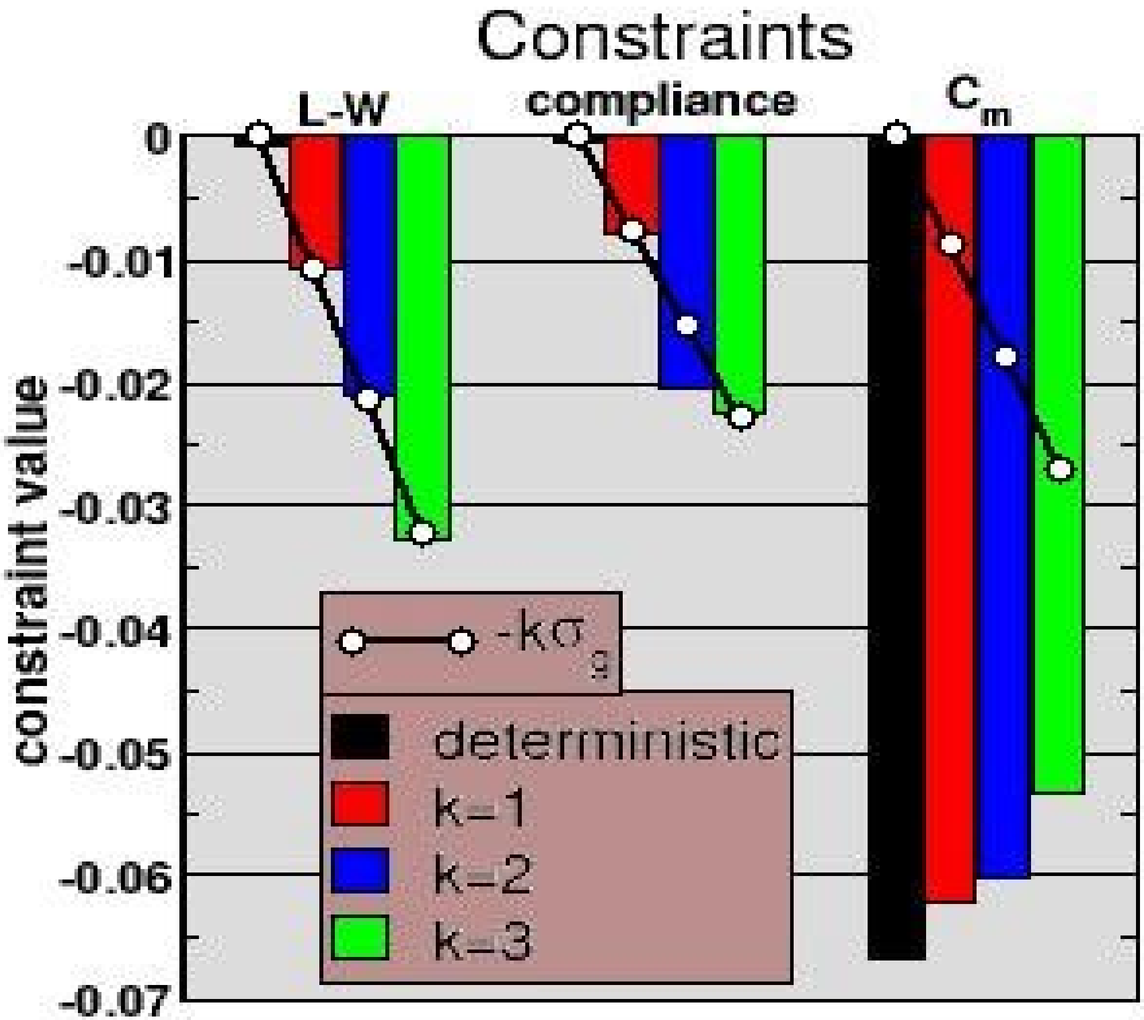
$M_\infty = 0.8, \alpha = 1^o$



# Four-DV Sample Results



# Four-DV Sample Results



# Concluding Remarks

- **Demonstrated UBD for MDO (Euler CFD and FEM) of 3-D wing in transonic flow**
- **Input uncertainty propagated by First-Order Second Moment method**
- **Simple implementation**
  - **established gradient-based optimization**
  - **quasi-analytic sensitivity derivatives**

slides: http://mdob.larc.nasa.gov/Conference/gumbert/AIAA02-2806.pdf
paper: http://mdob.larc.nasa.gov/Publications/Conference02.html