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Probabilistic Methods for Uncertainty Propagation Applied to Aircraft Design

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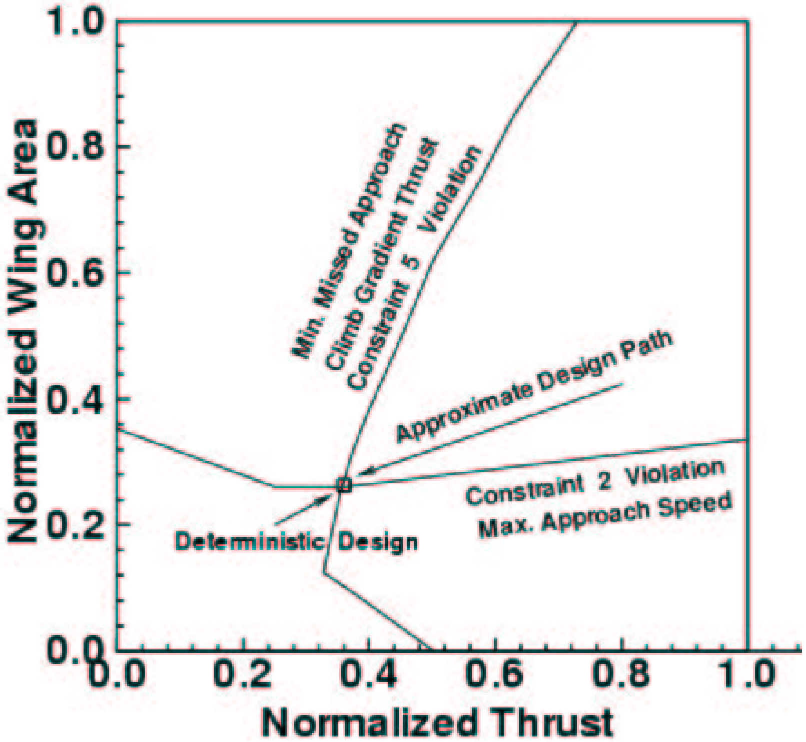
Introduction
FLOPS Code

- Widely distributed multidisciplinary computer program
- Aircraft analysis and conceptual / preliminary design
- Weights, aerodynamics, engine cycle, propulsion data scaling / interpolation, mission performance, takeoff / landing, noise footprint, and cost analyses + program control
- Analysis, parametric variation, optimization, and contour plotting

Introduction
FLOPS Code (Concluded)

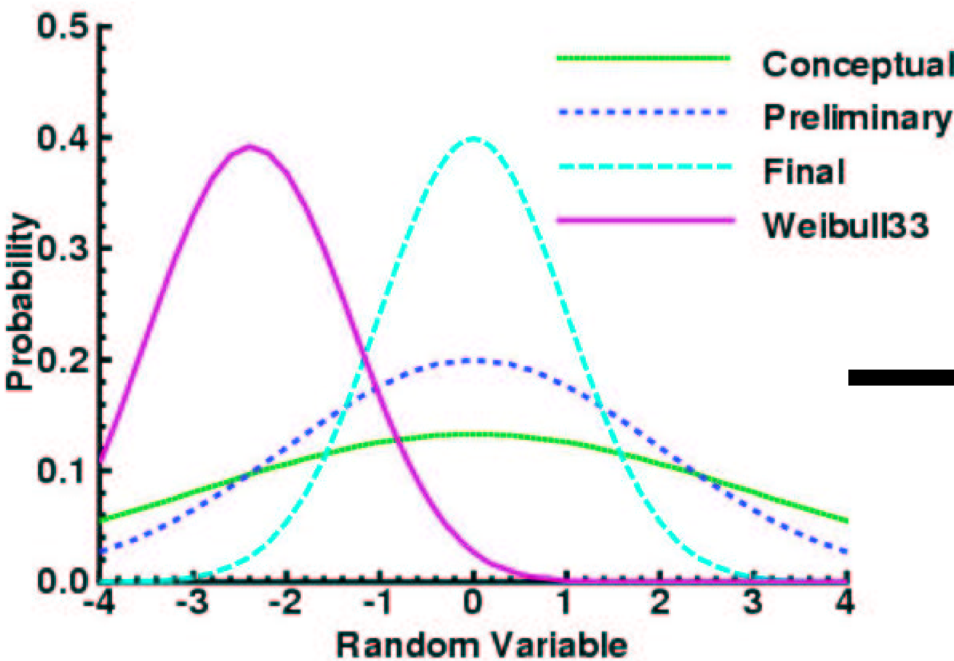
- Major code inputs: configuration, mission performance, and noise abatement design variables, plus externally-generated engine deck and aerodynamic drag data
- Major code outputs:
    - Analysis: performance, noise, weight, and cost metrics
    - Optimization: above + design objective and constraints
- Composite objective includes a highly nonlinear constraint violation penalty function
- Constraints = range, approach speed, takeoff & landing distances, approximate missed-approach and second-segment climb gradients
- Analysis module frequently fails to produce answer

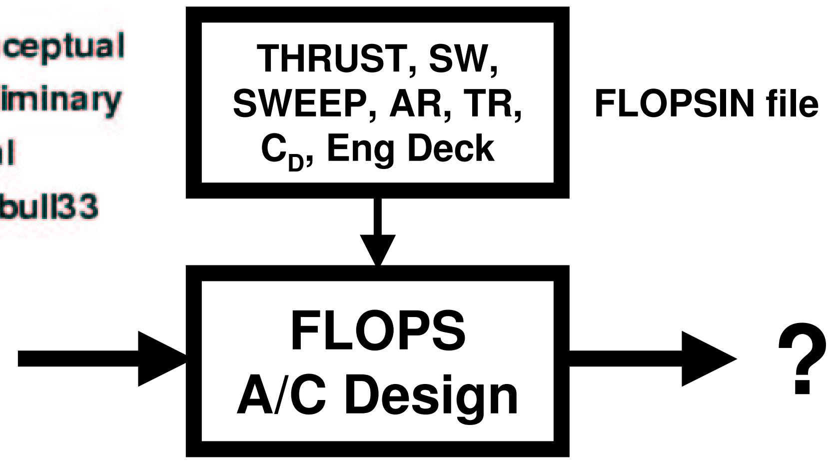
Introduction
FLOPS Subsonic Transport Design



- All inputs assumed to be precisely known (**deterministic**)
- Start from 5 design variable subsonic transport example case (xflp2.in)
- Modify input file so only THRUST (max thrust per engine) and SW (wing reference area) are active design variables
- Minimize aircraft takeoff weight with a fixed range
- Default BFGS optimization scheme
- Design path mostly in feasible region of design space
- Deterministic design lies at intersection of active constraints

Introduction
FLOPS Robust Design





- Characterize uncertainty (conceptual design = more uncertainty)
- Choose random variables (THRUST, SW); assess correlation
- Assign probability distribution function (PDF) type (**normal**, Weibull, etc.)
- Propagate input uncertainty through the process (FLOPS)
- Assess uncertainty in the output variables (**Obj**, Constraints, Weight, Cost, Noise)

Introduction
Uncertainty Methods

- Evaluate multidimensional probability density function (PDF) integral; determine associate cumulative density function (CDF)
  - Integrand not know in closed form
  - Limit state surface not known in closed form
- Uncertainty propagation methods approximate the integral evaluation
  - Method of moments – code modification uncertainty corrections
  - Monte Carlo simulation – code interaction with external software - define random input values
  - Commercial tools – code interaction with external software **UNIPASS**, ProFES, NESSUS

Methodology

Method of Moments Uncertainty Corrections

$$Traditional: \quad Obj^2_{uncertain} = \overline{Obj}^2_{deterministic} + \sigma^2_{Obj}$$

$$g_{uncertain} = \overline{g}_{deterministic} + K\sigma_g$$

$$FLOPS: \quad Obj_{uncertain} = \sqrt{\overline{Obj}^2_{deterministic} + \sigma^2_{Obj}}$$

$$g_{uncertain,m} = \overline{g}_{deterministic,m} + K\sigma_{g_m}, \quad m = 1,7$$

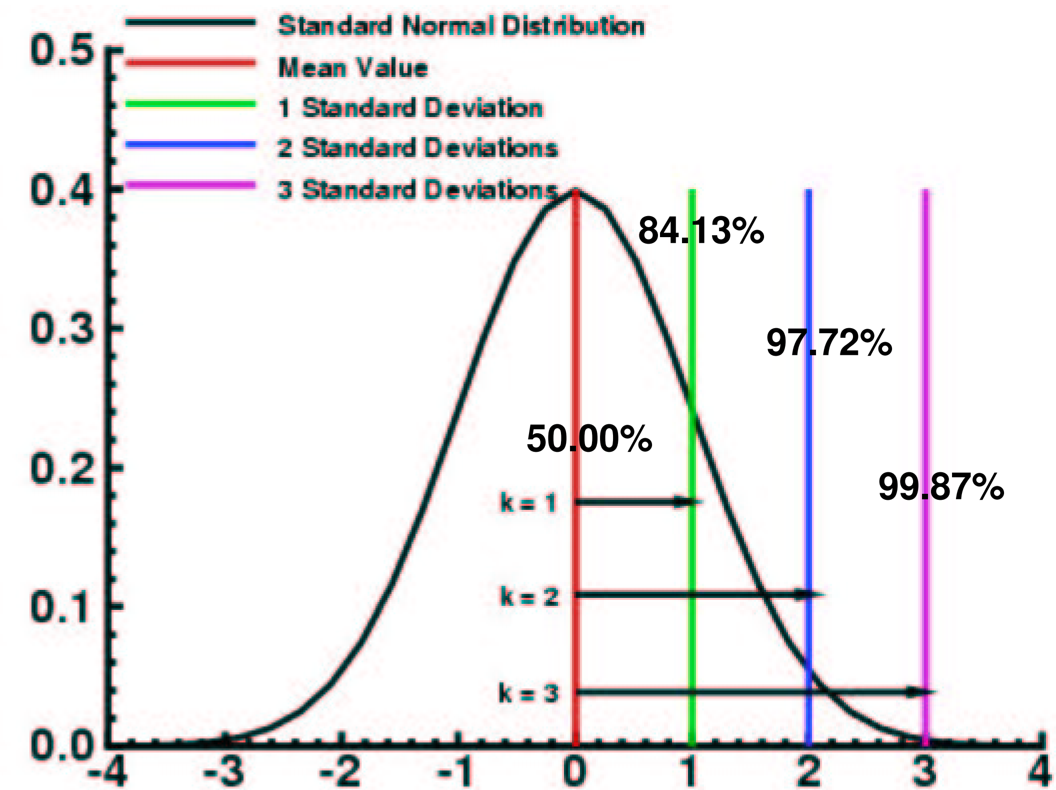
$$where: \quad \sigma^2_{Obj} = \sum_{i=1}^{n}\left(\frac{\partial Obj}{\partial b_i}\sigma_{b_i}\right)^2, \quad \sigma_{b_i} = \overline{b_i} * c.o.v._i$$

$$and: \quad \sigma_{g_m} = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial g_m}{\partial b_i}\sigma_{b_i}\right)^2}, \quad m = 1,7$$

$$c.o.v. = coefficient\ of\ variation$$

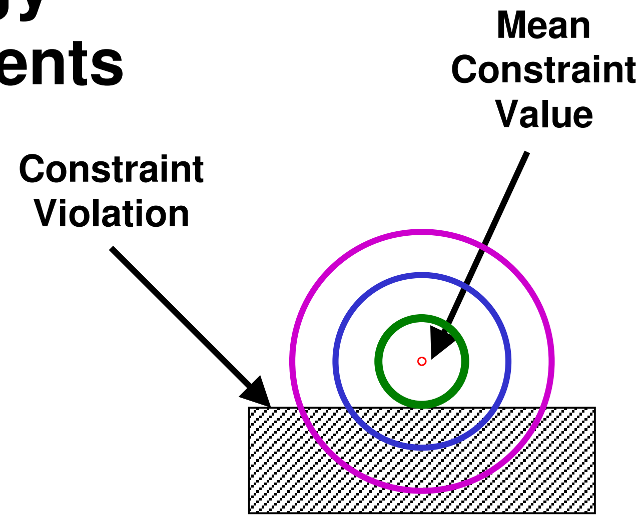
Derivatives constructed using Adifor
Automatic Differentiation tool applied to FLOPS code;
evaluated at mean values of random variables

Methodology
Method of Moments



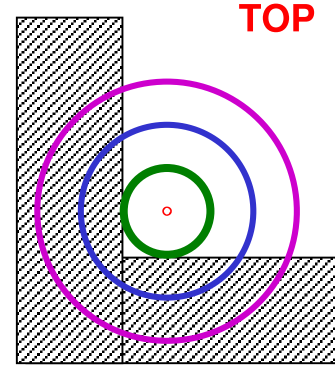
SIDE VIEW

Constraint mean value set back from constraint boundary for various K



1 Active Constraint
K = 1, 84.13% Constraint Satisfaction Probability

TOP VIEW



2 Active Orthogonal Constraints
K = 1, ~76.20% Constraint Satisfaction Probability

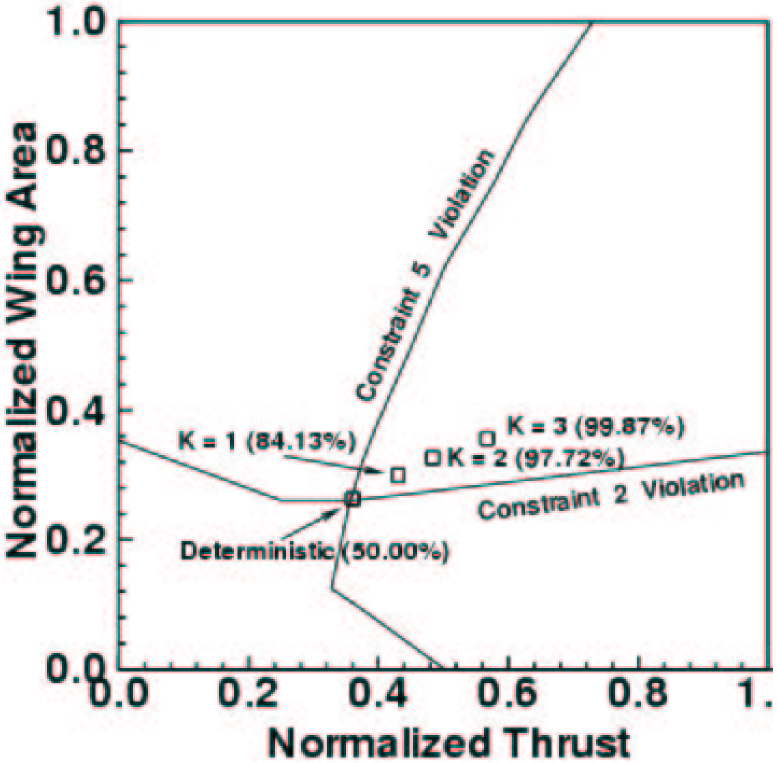
Methodology
UNIPASS version 4.2 Tool
from PredictionProbe, Inc.

- Many uncertainty modeling / propagation capabilities
- Capabilities used in this study include:
    - Interface to external code
    - Model uncertainties
    - Identify most probable point (MPP) via a computationally efficient non-gradient MPP simulation search method (SSM) for discontinuous design space
    - Compute reliability index (β) sensitivity data

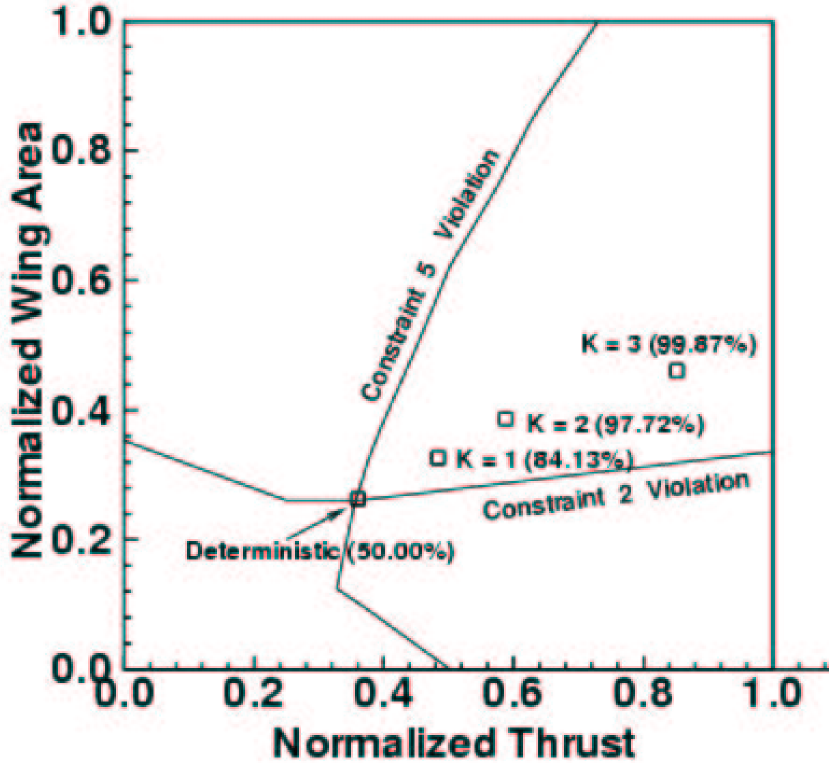
Methodology
FLOPS Uncertainty Propagation Demonstrations

- Test cases:
  - Subsonic transport aircraft design (xflp2.in)
    - Method of moments and Monte Carlo simulation
    - UNIPASS / SSM
  - Supersonic transport aircraft design (xflp3.in)
    - Method of moments / Monte Carlo simulation
  - THRUST and SW active / uncertain design variables
- Input variable uncertainty:
  - Subsonic transport aircraft design (c.o.v = 5% and 10%)
  - Supersonic transport aircraft design (c.o.v = 0.05% and 0.08%)
  - Assume normal distributions and uncorrelated variables

Method of Moments Results
Subsonic Transport
Deterministic and Robust Designs



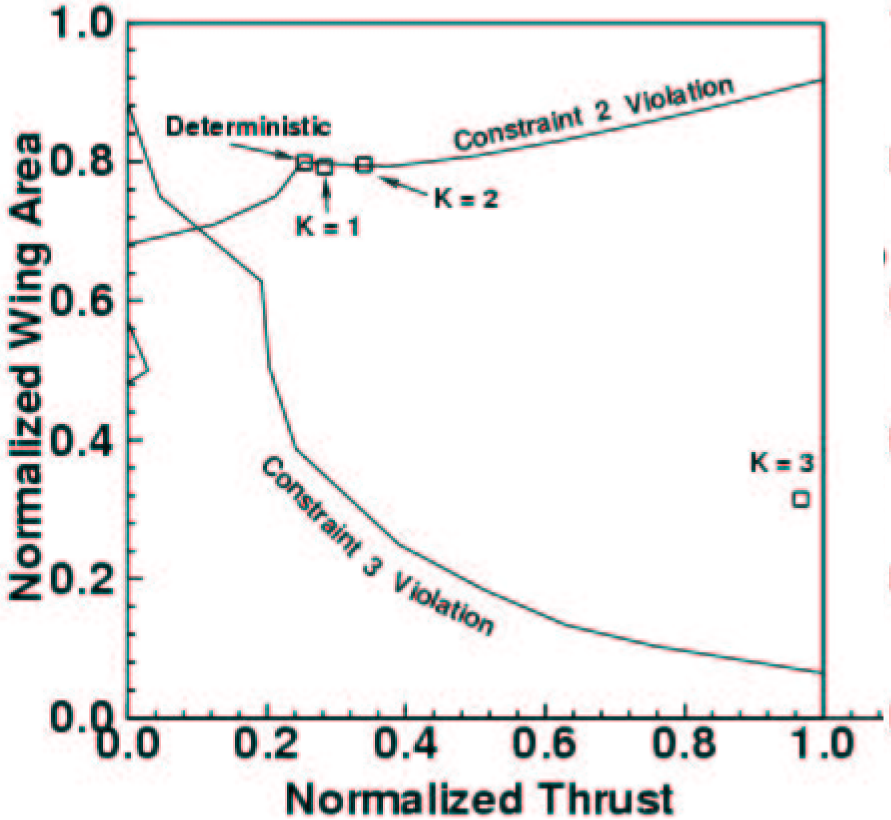
c.o.v. = 5%



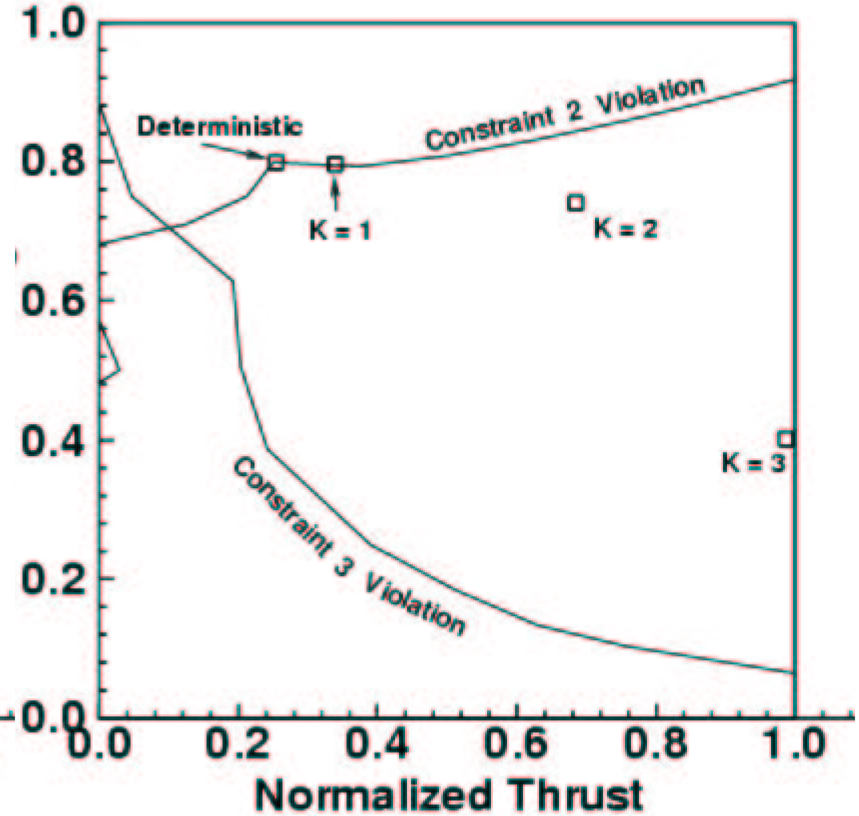
c.o.v. = 10%

Method of Moments Results

Supersonic Transport
Deterministic and Robust Designs



c.o.v. = 0.05%

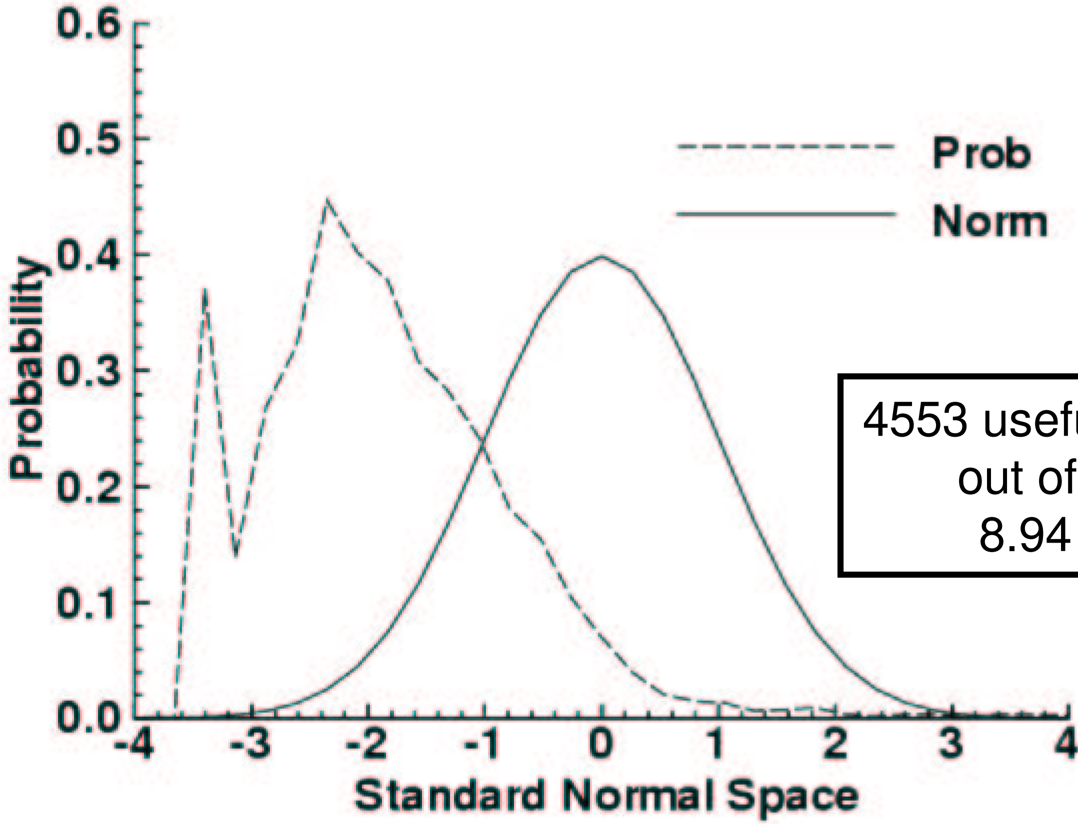


c.o.v. = 0.08%

Monte Carlo Simulation Results

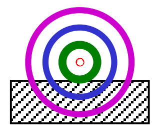
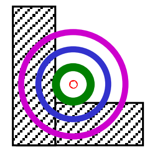
Supersonic Transport

Deterministic Design Point



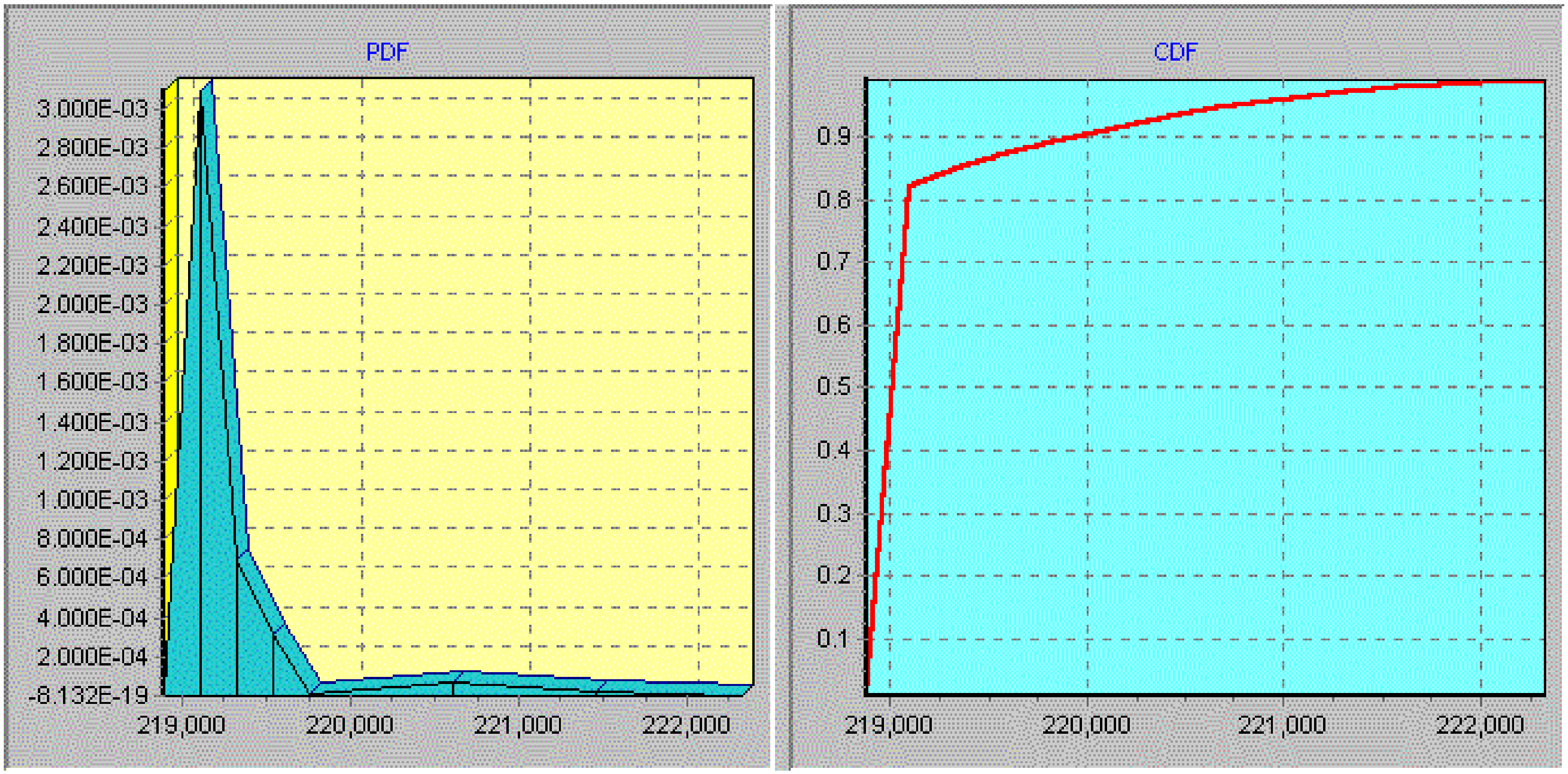
4553 useful FLOPS analyses
out of 5000 samples
8.94 % failure rate

Monte Carlo Simulation Results
Constraint Satisfaction Probability

Problem	Case	Constraint Satisfaction Probability (%)		
				Monte Carlo Simulation
Subsonic	Deterministic	50	~25	~30
Subsonic	K = 1	84	~76	~80
Subsonic	K = 2	98	~97	~90
Subsonic	K = 3	100	~100	~100
Supersonic	Deterministic	50	~25	~20
Supersonic	K = 1	84	~76	~30
Supersonic	K = 2	98	~97	~70
Supersonic	K = 3	100	~100	too close to DV bounds

Experimentally determined constraint satisfaction probabilities using Monte Carlo simulation with c.o.v. = 10% for all cases.

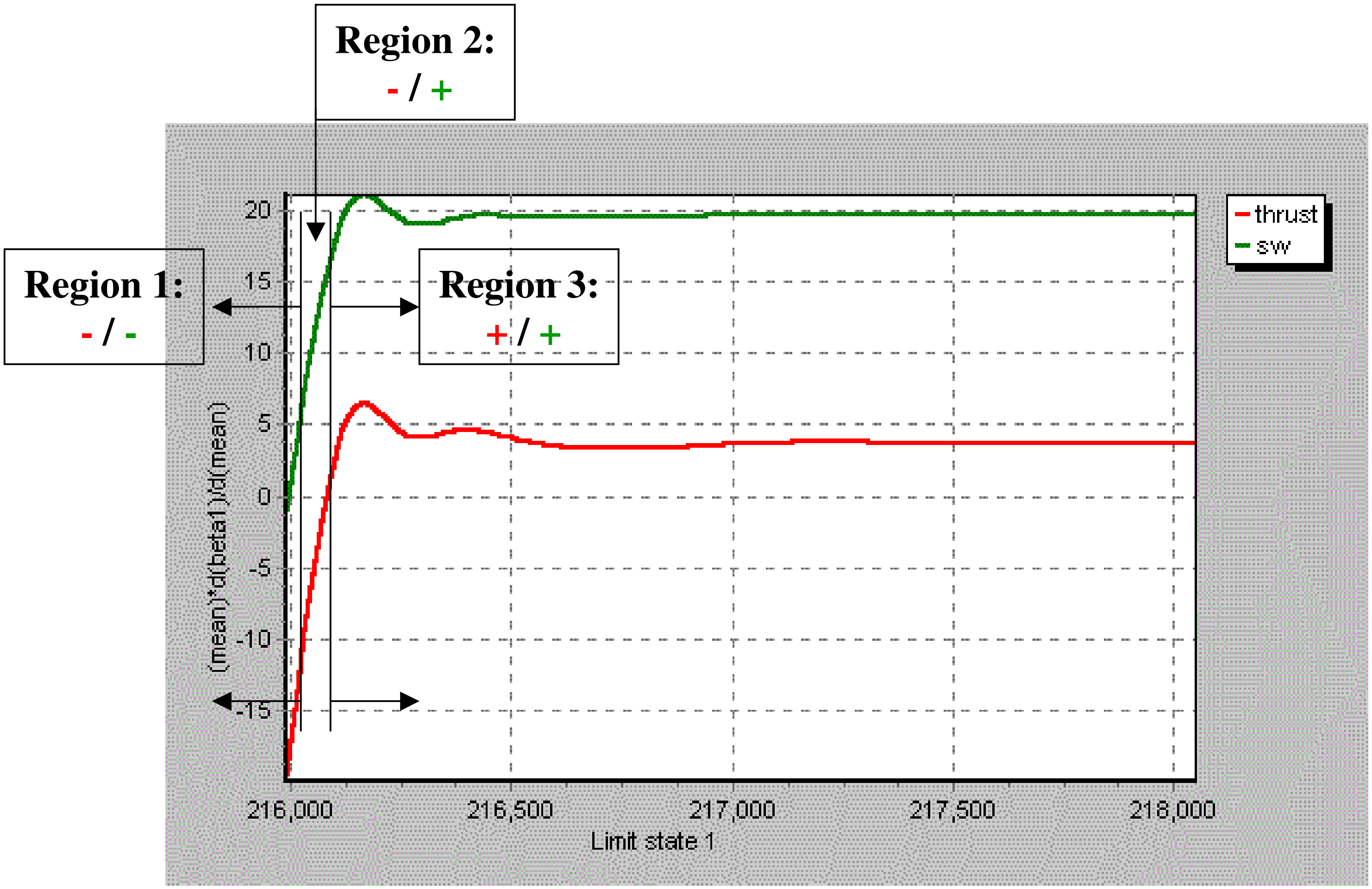
UNIPASS Results
PDF / CDF Analysis, Subsonic Transport
Deterministic Design Point, c.o.v = 5%



263 useful FLOPS analyses out of 457 samples; 42.45 % failure rate
Average FLOPS failure rate for analyses ~ 28%

UNIPASS Results

β-Gradient Analysis, Subsonic Transport
Deterministic Design Point, c.o.v = 5%



Results still under investigation

Concluding Remarks

- Initial FLOPS uncertainty propagation results
- Further research with code, methods, and test problems recommended
- Method of Moments uncertainty propagation
    - 2 example problems (subsonic / supersonic transport design)
    - 2 levels input uncertainty (c.o.v.) specified for each problem
    - 4 levels of satisfaction (*K*) required for a single constraint (0,1,2,3)
- Subsonic transport design problem
    - Similar, feasible deterministic / robust design paths
    - Reasonable levels of uncertainty imposed
    - Non-normal output distribution
    - Reasonably accurate constraint failure probability prediction
- Supersonic transport design problem
    - Different, curious, and infeasible deterministic / robust design paths
    - Great sensitivity to small levels of input uncertainty
    - Skewed, bi-modal output distribution
    - Constraint failure probability under-predicted
- UNIPASS / Simulation Search Method uncertainty propagation
    - Computationally more efficient than Monte Carlo simulation
    - PDF / CDF, MPP, and β-gradient computations demonstrated
    - Easily implement uncertainty propagation with external software

Methodology
Method of Moments

Random input variables: $B = \{b_1, \ldots, b_n\}$

Mean values: $\overline{B} = \{\overline{b_1}, \ldots, \overline{b_n}\},$

Standard deviations: $\sigma_b = \{\sigma_1, \ldots, \sigma_n\}, \quad \sigma_{b_i} = \overline{b_i} * (c.o.v.)_i\,,$

$$c.o.v. = coefficient\ of\ \ variation$$

Taylor series:

$$F(B) = F(\overline{B}) + \sum_{i=1}^{n} \frac{\partial F}{\partial b_i}(b_i - \overline{b_n}) + \frac{1}{2!}\sum_{j=1}^{n}\sum_{i=1}^{n} \frac{\partial^2 F}{\partial b_i \partial b_j}(b_i - \overline{b_n})(b_j - \overline{b_n})$$

Uncertainty Expressions (F=First, S=Second O=Order, M=Moment):

$$\overline{F} = F(\overline{B}), \quad FOFM$$

$$\sigma_F^2 = \sum_{i=1}^{n}\left(\frac{\partial F}{\partial b_i}\sigma_{b_i}\right)^2, \quad FOSM$$

$$\overline{F} = F(\overline{B}) + \frac{1}{2!}\sum_{i=1}^{n}\frac{\partial^2 F}{\partial b_i^{\,2}}\sigma_{b_i}^2, \quad SOFM$$

$$\sigma_F^2 = \sum_{i=1}^{n}\left(\frac{\partial F}{\partial b_i}\sigma_{b_i}\right)^2 + \frac{1}{2!}\sum_{j=1}^{n}\sum_{i=1}^{n}\left(\frac{\partial^2 F}{\partial b_i \partial b_j}\sigma_{b_i}\sigma_{b_j}\right)^2, \quad SOSM$$