MS7 and MS13

Optimization of Engineering Systems Governed by Differential Equations

Engineering Optimization with Variable-Fidelity Models

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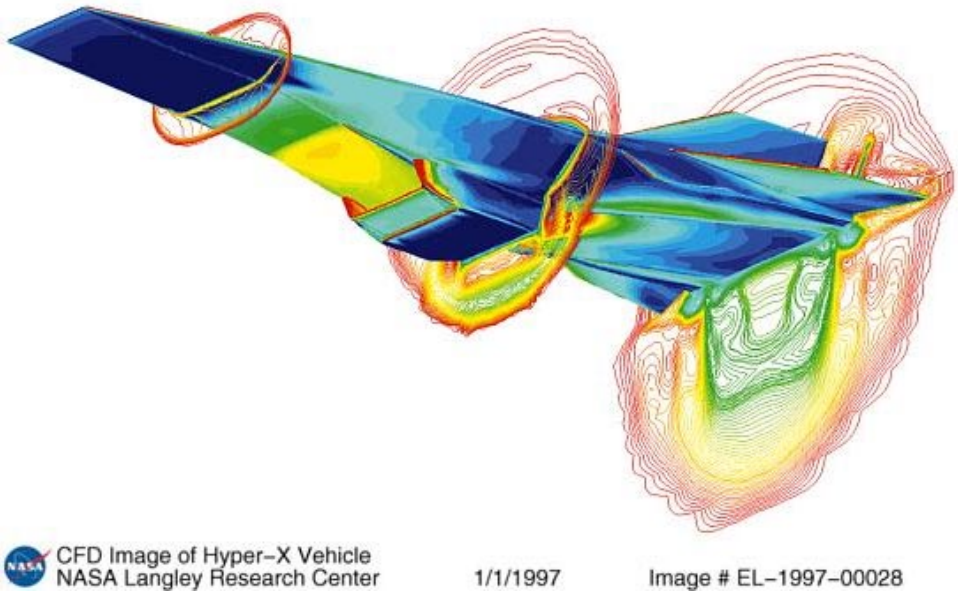
Outline

- **Overview of the area and MS7/MS13**
- **Engineering optimization with variable-fidelity models**
  - **The problem**
  - **First-order approximation/model management optimization (AMMO)**
  - **Some demonstration problems and results**
  - **Concluding remarks**

Optimization of Engineering Systems Governed by Differential Equations

X-33

Hyper-X



- **Engineering design optimization problem: Improve or optimize several objectives subject to satisfying a set of design and physical constraints from contributing disciplines (e.g., aerodynamics, structures, propulsion, etc.); some of the constraints are coupled sets of differential equations**

- **Other categories of problems: parameter estimation, optimal control**

Optimization of Engineering Systems Governed by Differential Equations, cont.

(see the current SIAG/OPT Views-and-News)

Single-Discipline Optimization

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x, u(x)) \\ \text{subject to} \quad & h(x, u(x)) = 0 \\ & g(x, u(x)) \geq 0 \\ & x_l \leq x \leq x_u, \end{aligned}$$

where, given x, $u(x)$ is computed via

$$A(x, u(x)) = 0$$

Multidisciplinary Optimization (MDO)

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & f(x, u(x)) \\ \text{subject to} \quad & h(x, u(x)) = 0 \\ & g(x, u(x)) \geq 0 \\ & x_l \leq x \leq x_u, \end{aligned}$$

where, given x, $u(x)$ is computed via $A(x, u(x)) =$

$$\begin{pmatrix} A_1(x, u_1(x), \ldots, u_N(x)) \\ \vdots \\ A_N(x, u_1(x), \ldots, u_N(x)), \end{pmatrix} = 0$$

Multidisciplinary analysis: physically consistent and thus meaningful design

Optimization of Engineering Systems Governed by Differential Equations, cont.

- **Difficulties specific to optimization subject to DE:**
  - **Expensive function and derivative evaluation (especially in MDO)**
  - **Problem formulation**
  - **Potentially large and/or nonsmooth problems**
- **MS7: Addressing the expense of function and derivative evaluation**
  - **Model building (Machiels, LLNL; Patera et al., MIT) and model management in optimization (Alexandrov, NASA Langley; Lewis, W&M)**
- **MS13: Optimization techniques and applications**
  - **Parameter estimation in models for pesticide transport (Bock et al., Heidelberg)**
  - **Ship design optimization (Campana et al., INSEAN)**
  - **Optimal control (Giesy et al., NASA Langley)**
  - **Model identification (Meade et al., Rice)**

Engineering Optimization with Variable-Fidelity Models

Solving

$$\begin{aligned} \textbf{minimize} \quad & f(x) \\ \textbf{subject to} \quad & h(x) = 0 \\ & g(x) \leq 0 \\ & x_l \leq x \leq x_u \end{aligned}$$

with expensive f, g, h: evaluate via high-fidelity or high-resolution analyses

- **Minimize expense of using in single-discipline optimization and MDO (additional benefit - easier integration and interactive MDO)**
- **Heuristic approximate techniques long in use in engineering optimization**
- **Present work: Systematic model management that guarantees convergence to high-fidelity results with minimal high-fidelity evaluations (see publications on the MDOB web page for references)**

1st Order Approximation/Model Management Optimization (AMMO) Framework

- **Given a suite models of a physical phenomenon**
  - **Approximations based on data fits (polynomial response surfaces, kriging approximations, splines, etc.)**
  - **Variable resolution (a single model evaluated on meshes of varying refinement)**
  - **Variable-fidelity physics models (e.g., Euler equations vs. Navier-Stokes equations)**
- **Idea**
  - **For any optimization subproblem, replace the local Taylor series model with an arbitrary model that satisfies first-order consistency conditions with respect to a model designated as "truth"**
- **Can be imposed on any algorithm, in principle (e.g., Alexandrov and Lewis, 1996)**

An Example of a 1st Order AMMO Algorithm

Initialize x_c, Δ_c

Do until convergence:

Select model a_c **with** $a_c(x_c) = f(x_c)$; $\nabla a_c(x_c) = \nabla f(x_c)$

Solve approximately for $s_c = x - x_c$:

$$\begin{aligned} \underset{s}{\text{minimize}} \quad & a_c(x_c + s) \\ \text{subject to} \quad & l \leq x \leq u \\ & \| s \|_\infty \leq \Delta_c \end{aligned}$$

Compute $\rho_c \equiv \frac{f(x_c) - f(x_c + s_c)}{f(x_c) - a_c(x_c + s_c)}$

Accept s_c **if** $f(x_c) > f(x_c + s_c)$; **otherwise reject**

Update Δ_c

End do

$\rho_c \leq 10^{-5}$	**Reduce** Δ_c
$10^{-5} < \rho_c \leq 0.6$	**Leave** Δ_c **unchanged**
$\rho_c > 0.6$	**Increase** Δ_c

Enforcing 1st Order Consistency Is Easy

$$a_c(x_c) = f(x_c)$$

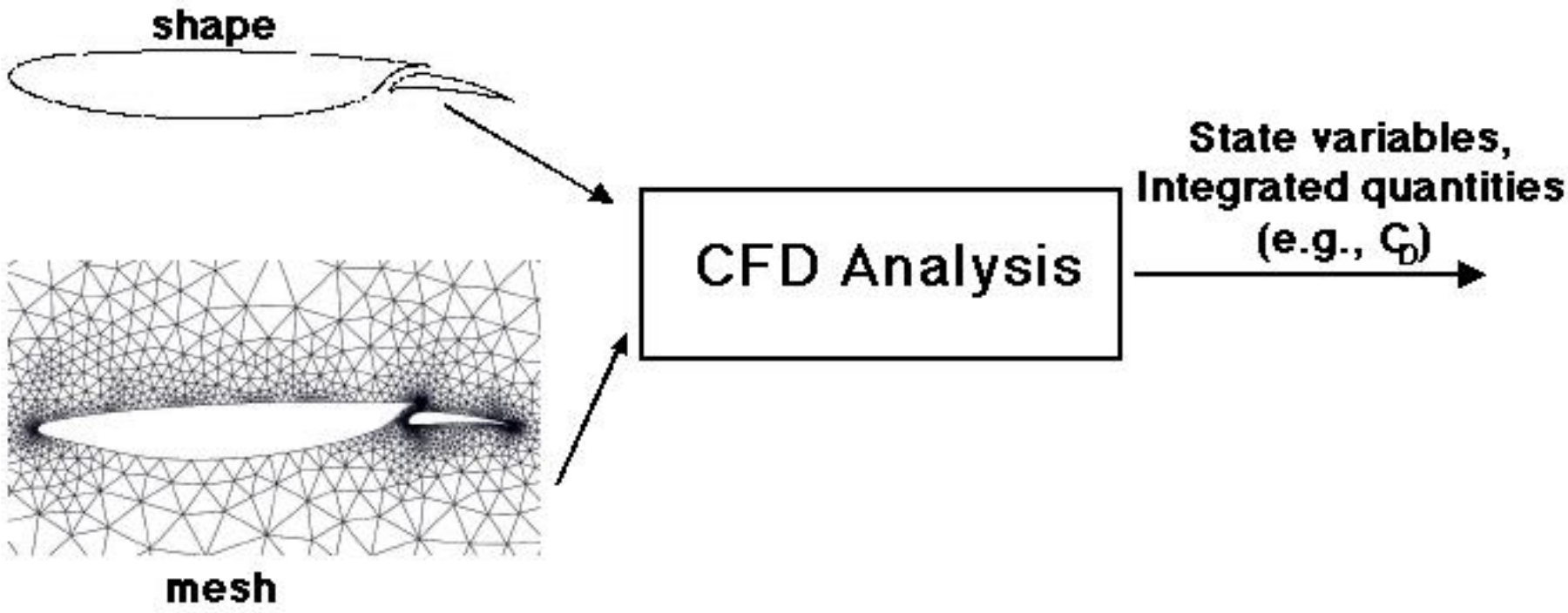
$$\nabla a_c(x_c) = \nabla f(x_c)$$

- **Can be relaxed to zero-order consistency theoretically**
- **Are easily enforced (Chang, Haftka, Giles, Kao, 1993):**
  - **Given $f_{hi}(x)$ and $f_{lo}(x)$, define $\beta(x) \equiv \frac{f_{hi}(x)}{f_{lo}(x)}$**
  - **Given x_c, build $\beta_c(x) = \beta(x_c) + \nabla\beta(x_c)^T(x - x_c)$**
  - **Then $a_c(x) = \beta_c(x) f_{lo}(x)$ satisfies the consistency conditions**

Convergence vs. Performance

- **Convergence**
  - **Need only a notion of two models, one arbitrarily assigned "high fidelity" or "truth", the other - "low fidelity"**
  - **1st-order consistency mimics the local Taylor series model behavior**
  - **Consistency conditions + standard assumptions for a particular optimization algorithm + boundedness assumptions on the 2nd-order lo-fi model information $\Longrightarrow$ global convergence to hi-fi critical points or solutions**
  - **For general NLP, SQP-based AMMO is our currently preferred algorithm**
- **Performance**
  - **Problem/model dependent; depends on the predictive properties of the lo-fi model wrt improvement characteristics of the hi-fi model**
  - **Worst-case scenario: conventional optimization for NLP**
  - **Now looking into the behavior of the correction for analytical predictions of performance**

Demonstration Problems: Aerodynamic Optimization



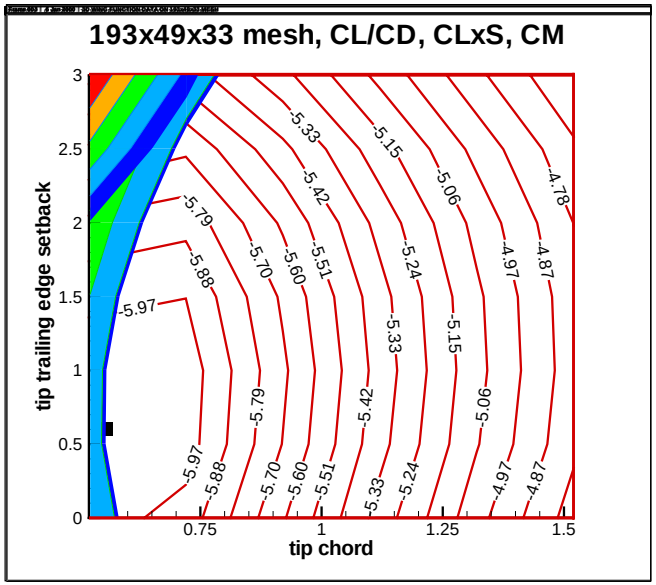
minimize Integrated quantities, such as $-\frac{L}{D}$ or C_D

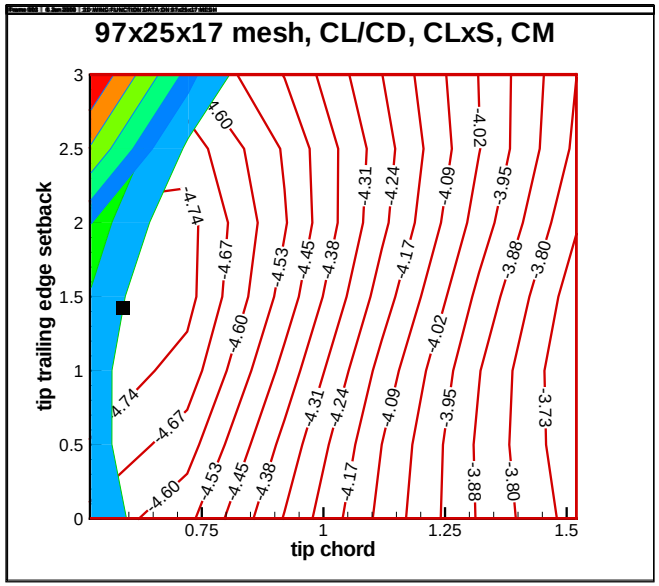
subject to constraints on, e.g., pitching and rolling moment coefficients, etc.

$x_l \leq x \leq x_u$

A Demonstration Problem: Aerodynamic Optimization of a 3D Wing (AIAA-2000-0841, Alexandrov, Lewis, Gumbert, Green, Newman)

- **Problem: minimize $-\frac{L}{D}$ s.t. nonlinear constraints**
- **Variable-resolution models: Euler equations on a variety of meshes**
- **Algorithms: based on SQP, augmented Lagrangian, a multilevel approach**





- **Threefold improvement in terms of hi-fi function and derivative evaluations across all AMMO algorithms tested**

Current Demonstration Problem: Multi-Element Airfoil

(AIAA-2000-4886, Alexandrov, Nielsen, Lewis, Anderson)

- **A two-element airfoil designed to operate in a transonic regime — inclusion of viscous effects is very important**
- **Governing equations: time-dependent Reynolds-averaged Navier-Stokes**

$$A\frac{\partial Q}{\partial t} + \oint_{\partial\Omega} \vec{F_i} \cdot \hat{n} dl - \oint_{\partial\Omega} \vec{F_v} \cdot \hat{n} dl = 0,$$

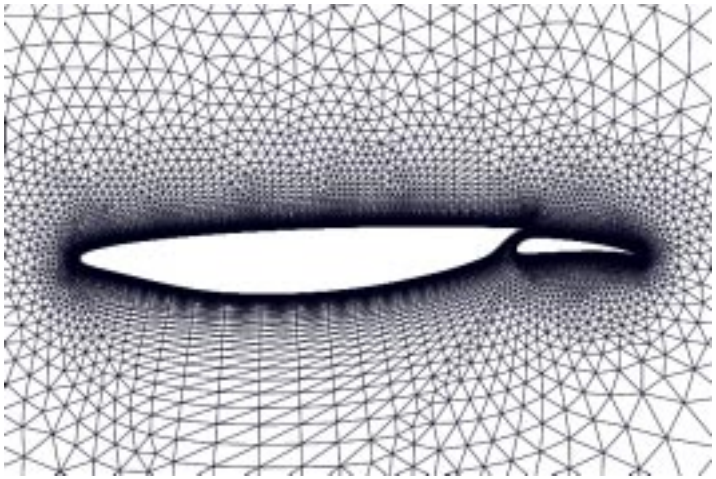
  where $\vec{F_i}$ and $\vec{F_v}$ are the inviscid and viscous fluxes, respectively

- **Flow solver (FUN2D) – unstructured mesh methodology (Anderson, 1994)**
- **Sensitivity derivatives – hand-coded adjoint approach (Anderson, 1997)**
- **Conditions:**
  - $M_\infty = 0.75$
  - $Re = 9 \times 10^6$
  - $\alpha = 1^\circ$ **(global angle of attack)**

Current Demonstration Problem, cont.

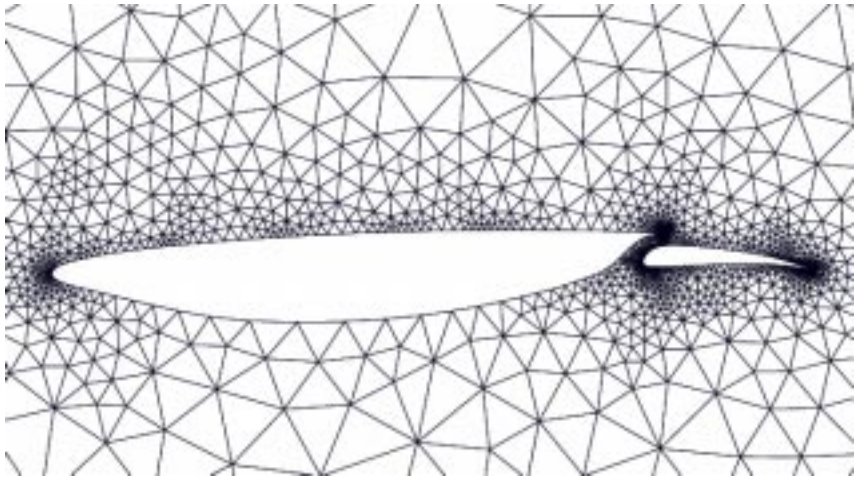
- **Hi-fi model – FUN2D analysis in RANS mode**
- **Lo-fi model – FUN2D analysis in Euler mode**
- **Computing on SGI Origin^{TM} 2000, 4 R1OK processors**

Viscous mesh:
10449 nodes and 20900 triangles



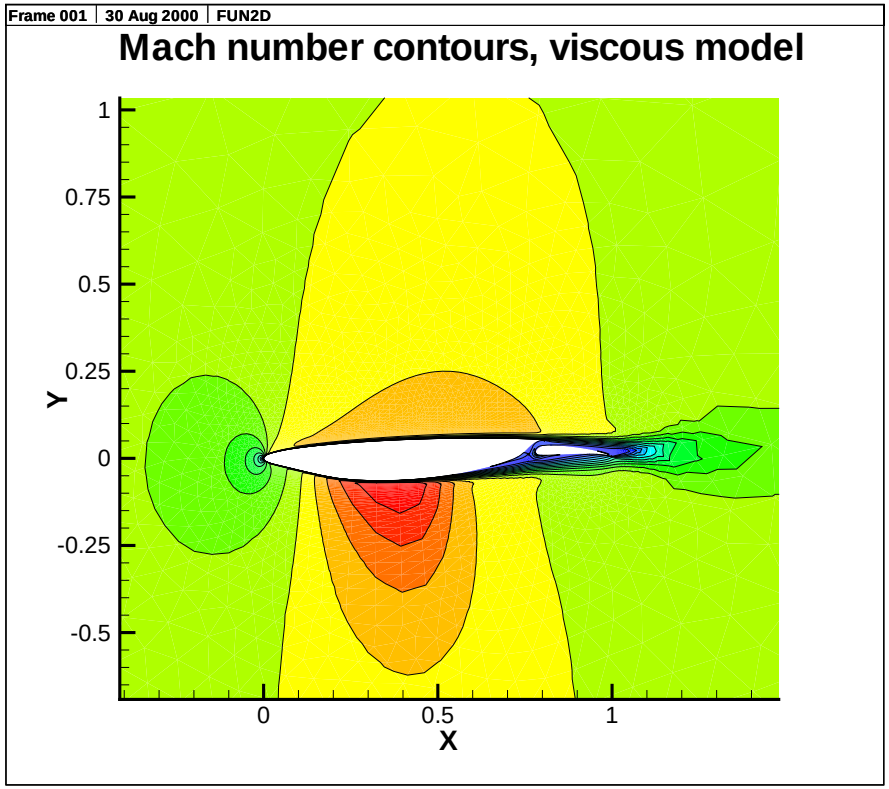
t/analysis $\approx$ 21 min
t/sensitivity $\approx$ 21 or 42 min

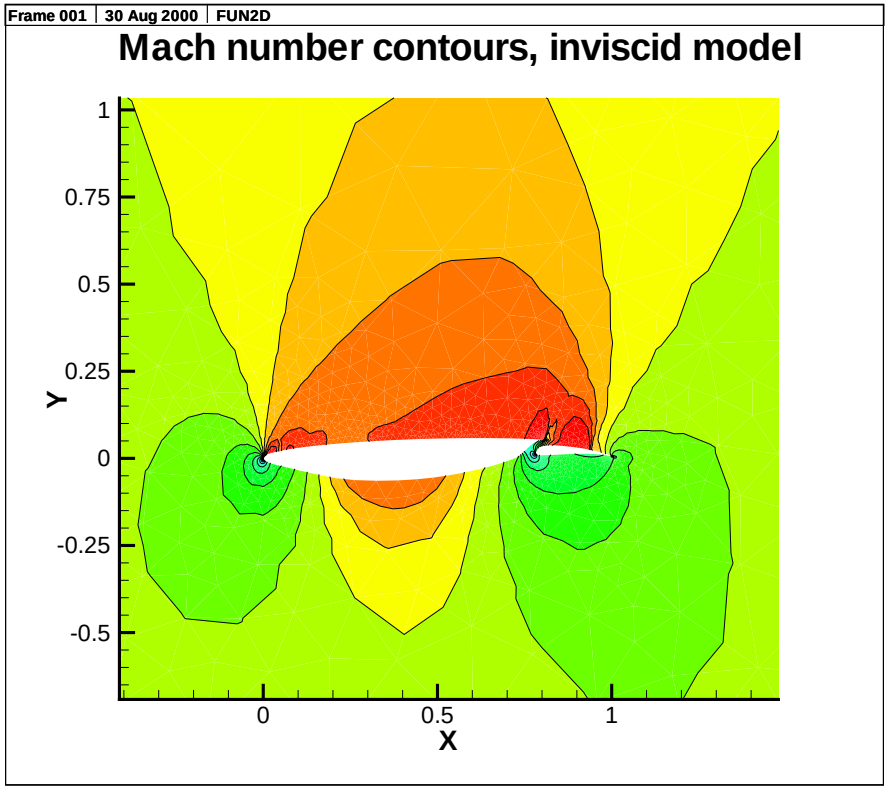
Inviscid mesh:
1947 nodes and 3896 triangles



t/analysis $\approx$ 23 sec
t/sensitivity $\approx$ 100 or 77 sec

Viscous Effects





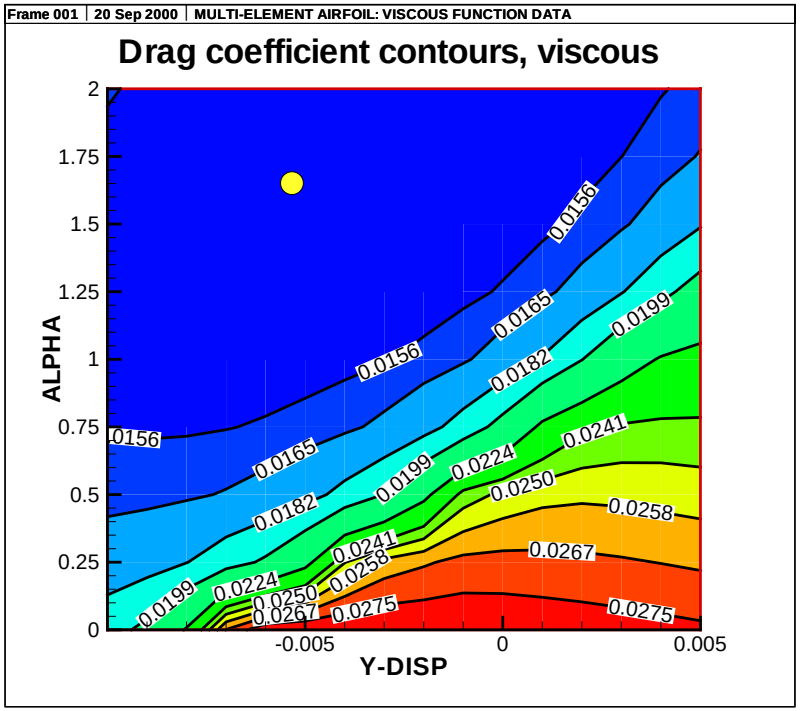
- **Boundary and shear layers are visible in the viscous case.**

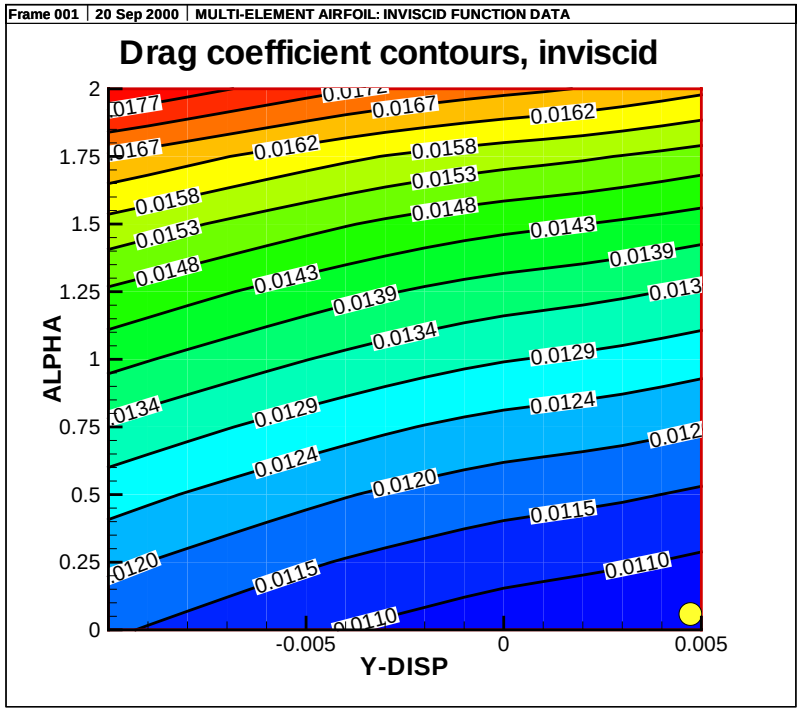
Computational Experiments

- **Objective function:** minimize drag coefficient
- **Case 1:** (for visualization)
  - Variables: angle of attack, y-displacement of the flap
  - Solve problem with hi-fi evaluations alone using a commercial optimization code (PORT, Bell Labs)
  - Solve the problem with AMMO, PORT used for lo-fi subproblems
- **Case 2:**
  - Variables: angle of attack, y-displacement of the flap, geometry description of the airfoil; 84 variables total
  - Same experiment

Models

- **Time/function for inviscid model negligible compared to viscous model**
- **However, their descent trends are reversed**

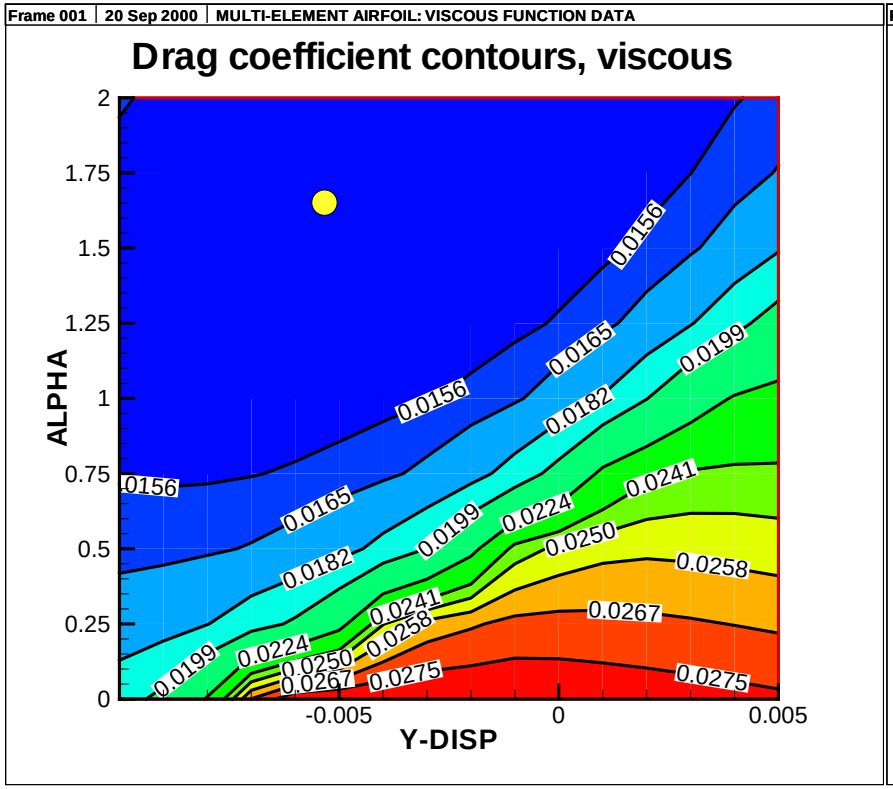


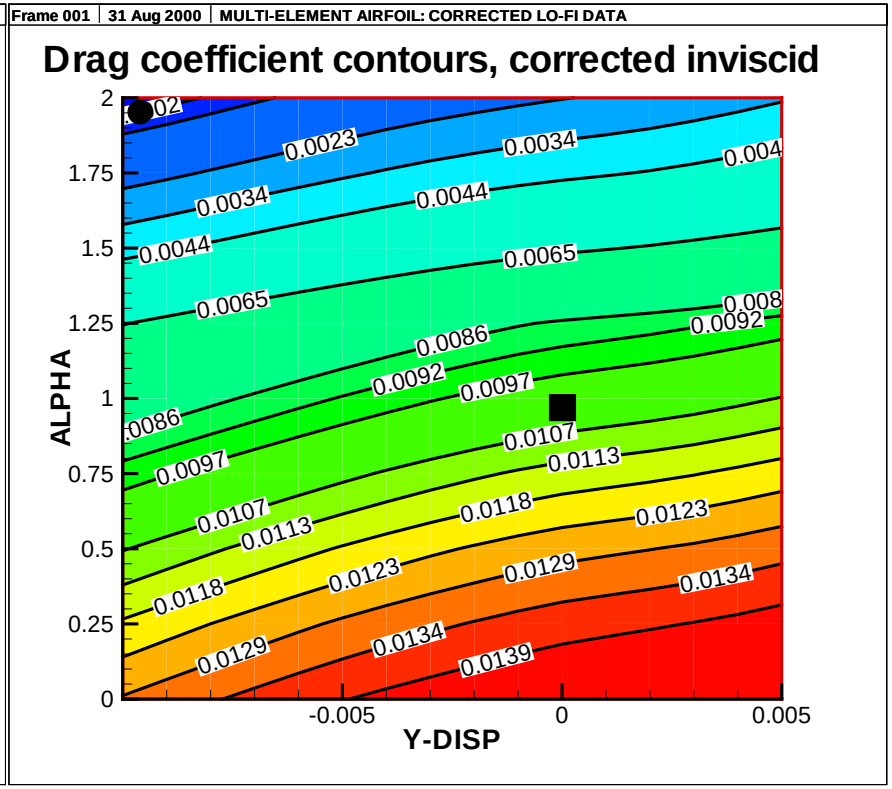


Example: AMMO Iteration History with 2 variables (surrogate functions)

Iteration 1. Starting point: $\alpha = 1.0$, y-disp $= 0.0$

High-fidelity objective vs. corrected low-fidelity objective





New point: $\alpha = 2.0$, y-disp $= -0.01$

Example: AMMO Iteration History, cont.

- **Similar effect in the next iteration**
- **Solution ($\alpha = 1.6305°$, flap y-displacement $= -0.0048$) located at iteration 2**
- $C_D^{\text{initial}} = 0.0171$ **at ($\alpha = 1°$, flap y-displacement $= 0$)**
- $C_D^{\text{final}} = 0.0148$**, a decrease of approximately 13.45%.**

Performance Summary

Notation: No. functions / No. Gradients

Test	hi-fi eval	lo-fi eval	total t	factor
PORT with hi-fi analyses, 2 var	14/13		$\approx$ 12 hrs	
AMMO, 2 var	3/3	19/9	$\approx$ 2.41hrs	$\approx$ 5
PORT with hi-fi analyses, 84 var	19/19		$\approx$ 35 hrs	
AMMO, 84 var	4/4	23/8	$\approx$ 7.2hrs	$\approx$ 5

Concluding Remarks

- **AMMO has yielded fivefold improvement**
- **Enforcing model consistency via Chang et al. correction works very well**
- **Derivative information is crucial when a lower-fidelity model does not capture high-fidelity descent information**
- **For large problems, variable-fidelity physics modeling may be recommended**
- **Current work**
  - **Considering incorporation of *a posteriori* error estimates into AMMO (with Patera et al.)**
  - **Considering larger problems**
  - **Considering other models**