Optimization with Variable-Fidelity Models Applied to Wing Design

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Outline

- **The problem and motivation**
- **Related research**
- **Variable-fidelity models**
- **SQP-based AMF**
- **Computational demonstration**
- **Conclusions to-date**
- **Ongoing work**

The Problem

$$\begin{array}{ll} \textbf{minimize} & f(x) \\ \textbf{subject to} & g(x) \leq 0 \\ & l \leq x \leq u, \end{array}$$

with expensive f and g: high-fidelity or high-resolution analysis required

- **Address expense of repeated use of high-fidelity models in MDO**
  - **Solutions of coupled PDE typically required at each iteration (function evaluations increase for uncoupled formulations)**
  - **Use of lower-fidelity models alone does not guarantee improvement in higher-fidelity design**
- **Allow for easier integration and interactive design in MDO**
- **Demonstrate feasibility of proposed methods on engineering problems**

Related Research

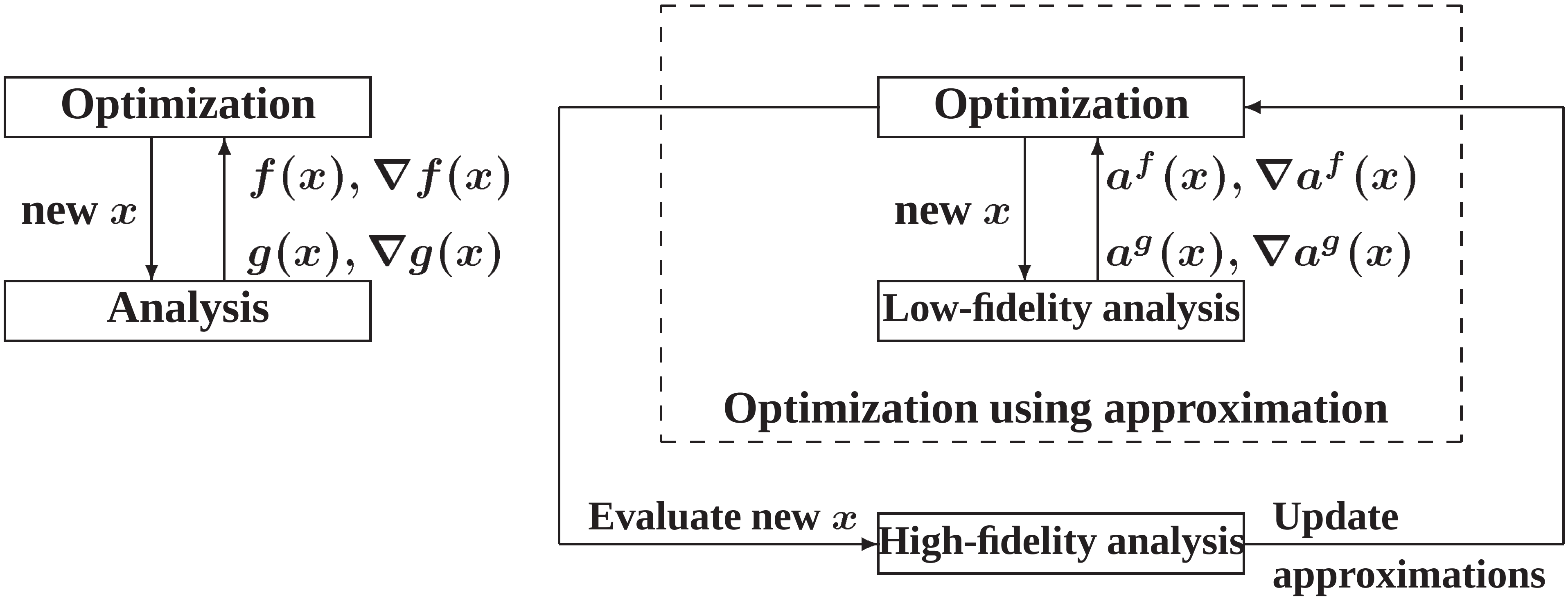
Variable- delity computational models used in engineering for a long time

Survey on use of approximations in structural optimization, Barthelemy and Haftka, 1993

Some related work conducted or supported at NASA Langley

– Partially converged models, Gumbert et al., Ta'asan et al. (ICASE)
– Reduced-order models, Silva et al. (NASA LaRC), Patera et al. (MIT)
– A posteriori error bounds for outputs of PDE and sensitivity derivatives of outputs, Lewis (ICASE), Patera et al. (MIT)
– Global/local optimization, Haftka et al. (U. Florida)
– Managing models/approximations in optimization, Alexandrov et al. (NASA LaRC)

Conventional Optimization vs. Approximation Management Framework (AMF)



Variable-Fidelity Computational Models in CFD

- Approximations based on fitting response surfaces to sampled data
  - Polynomials (RSG package, Kaufman et al.)
  - Kriging (coded locally)
  - Splines (PORT package, Gay et al.)
- Variable-resolution models
  - Single analysis performed on meshes of varying refinements
  - Assume: the finer the mesh, the higher the model fidelity
- Variable-fidelity physics models
  - E.g., Euler equations vs. Navier-Stokes equations

First-Order AMFs for Constrained Optimization

AMF = underlying optimization algorithm
+ systematic alternation of models + globalization

- **"First-Order" means sensitivities are used**
- **We have studied three AMFs based on**
  - **Augmented Lagrangian approach**
  - **Sequential Quadratic Programming (SQP)**
  - **A multilevel optimization algorithm (MAESTRO)**
- **Current objective**
  - **Demonstrate methodology on increasingly realistic engineering problems, in single-discipline optimization and MDO contexts**
  - **Identify the most promising AMF in both contexts**

The Underlying Optimization Algorithm for SQP-Based AMF

- x_c — the current iterate, x_+ — next iterate
- B_c — an approximation to $\nabla^2 f(x_c)$
- $P(x;\mu) \equiv f(x) + \sum_{i=1}^{m} \mu_i \max[0, -g_i(x)]$, $m =$ # of constraints

Initialize x_c, μ_c

Do until convergence:

Solve the following subproblem for $s_c = x - x_c$:

$$\begin{array}{ll} \underset{s}{\text{minimize}} & \nabla f(x_c)^T s + \frac{1}{2} s^T B_c s \\ \text{subject to} & g(x_c) + \nabla g(x_c)^T s \leq 0 \\ & l \leq x_c + s \leq u \end{array}$$

Set $x_+ = x_c + \alpha_c s_c$ with α_c s.t. $P(x_+;\mu_c) < P(x_c;\mu_c)$

Update μ_c

End do

SQP-AMF

Initialize x_c, μ_c, Δ_c; **compute** $P(x_c)$

Do until convergence:

Select models a_c^f **and** a_c^g, **with**

$a_c^f(x_c) = f(x_c)$; $\nabla a_c^f(x_c) = \nabla f(x_c)$ **and**

$a_c^g(x_c) = g(x_c)$; $\nabla a_c^g(x_c) = \nabla g(x_c)$

Solve approximately for $s = x - x_c$:

$$\begin{array}{ll} \underset{s}{\textbf{minimize}} & a_c^f(x_c + s) \\ \textbf{subject to} & a_c^g(x_c) + \nabla a_c^g(x_c)^T s \le 0 \\ & l \le x \le u \\ & \| s \|_\infty \le \Delta_c \end{array}$$

Compute $P(x_c + s_c)$

Update Δ_c **and** x_c, μ_c **based on** $P(x_c)$ **vs.** $P(x_c + s_c)$

End do

Properties of SQP-AMF

- **Convergence**
  - **Strong convergence properties of the underlying algorithm: simple decrease in the merit function assures global convergence**
  - **Decrease in the merit function easily enforced in practice**
- **Inequality constraints handled directly**
- **Commercial software can be used for internal iterations**
- **Can arrange to start with solution of low-fidelity problem**
- **Easy to implement**

Enforcing the Consistency Conditions

$$a_c^f(x_c) = f(x_c); \nabla a_c^f(x_c) = \nabla f(x_c)$$

$$a_c^g(x_c) = g(x_c); \nabla a_c^g(x_c) = \nabla g(x_c)$$

- **Can be relaxed to zero-order consistency**
- **Are easily enforced (Chang et al. ’93):**
  - **Given $f_{hi}(x)$ and $f_{lo}(x)$, define $\beta(x) \equiv \frac{f_{hi}(x)}{f_{lo}(x)}$**
  - **Given x_c, build $\beta_c(x) = \beta(x_c) + \nabla\beta(x_c)^T(x - x_c)$**
  - **Then $a_c(x) = \beta_c(x) f_{lo}(x)$ satisfies the consistency conditions**

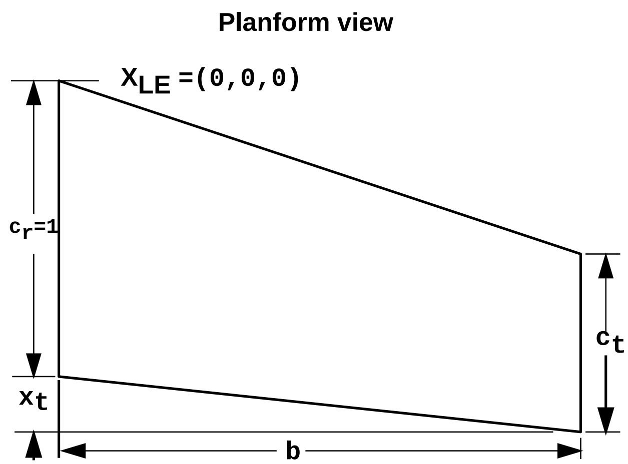
Computational Demonstration

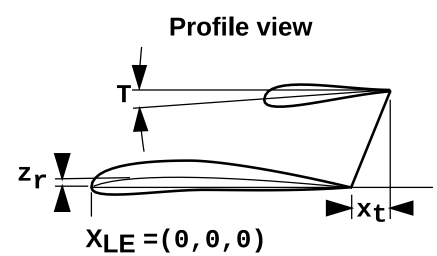
- Demonstrate feasibility on single-discipline, aerodynamic optimization problems with a small number of design variables
- Variable-resolution models represented by evaluating functions on meshes of varying refinements
- Approximations used as substitutes for functions to reduce cost and to simulate various model combinations (not used in conventional way)
- Computational experiment:
  - Solve high-fidelity problem with a well-known optimizer (e.g., NPSOL)
  - Solve problem with AMFs (SQP, Aug-Lagr, MAESTRO)
  - Compute number of "equivalent" high-fidelity function evaluations function evaluations for AMF to evaluate AMF's performance

3D Wing Optimization: Problem Description

Problem formulated and assembled by C.R. Gumbert

- **Analysis: Euler (NS/Euler code CFL3D, Rumsey et al., NASA LaRC)**
- **Conditions:** $M_\infty = 0.6, \alpha = 3.0$
- **Design variables: tip chord, tip trailing edge setback**

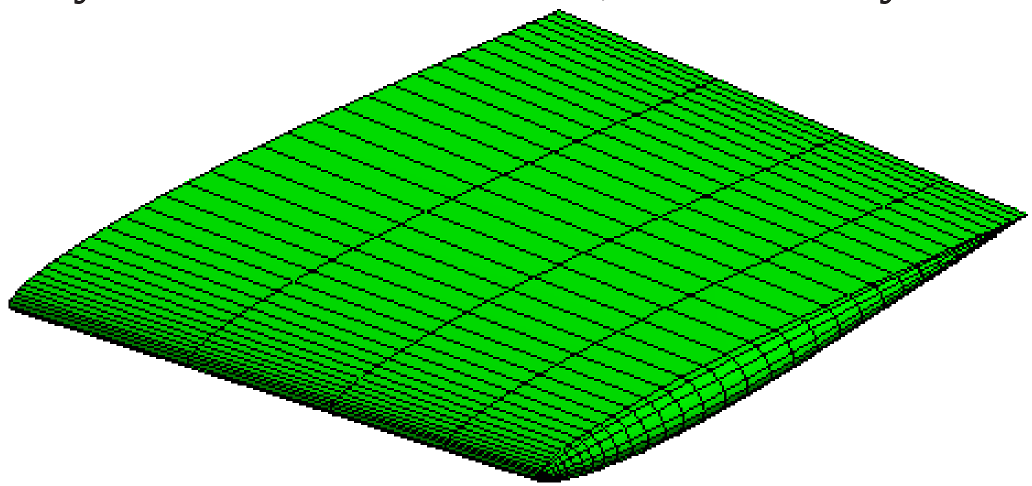




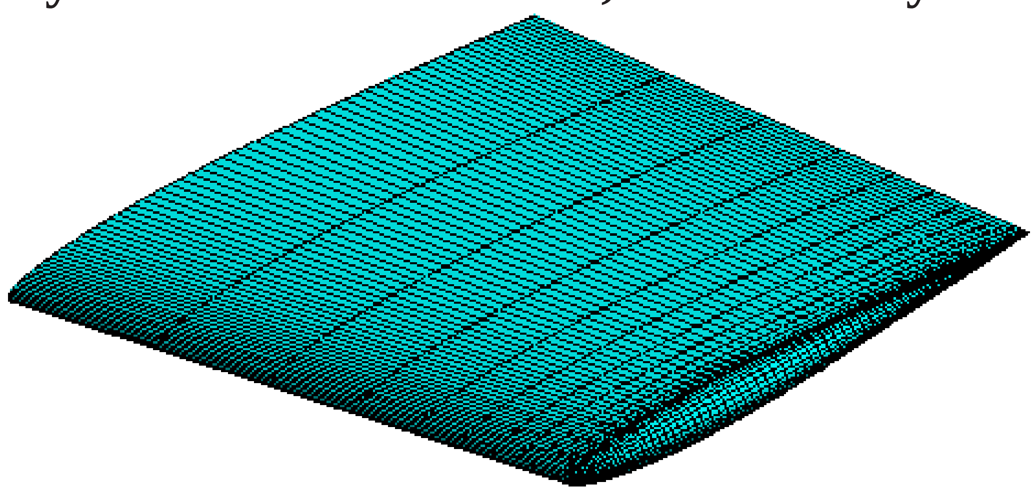
- **Objective:** $-\frac{L}{D}$
- **Constraints in lieu of multidisciplinary constraints: a lower bound on total lift $C_L S$, upper bounds on the pitching moment coefficient C_M and the rolling moment coefficient C_l**

3D Wing Optimization: Problem Description

Low-fidelity: analysis on 97x25x17 mesh, 8 min/analysis on Sun SPARC 1:



High-fidelity: analysis on 193x49x33 mesh, 64 min/analysis on Sun SPARC 1:



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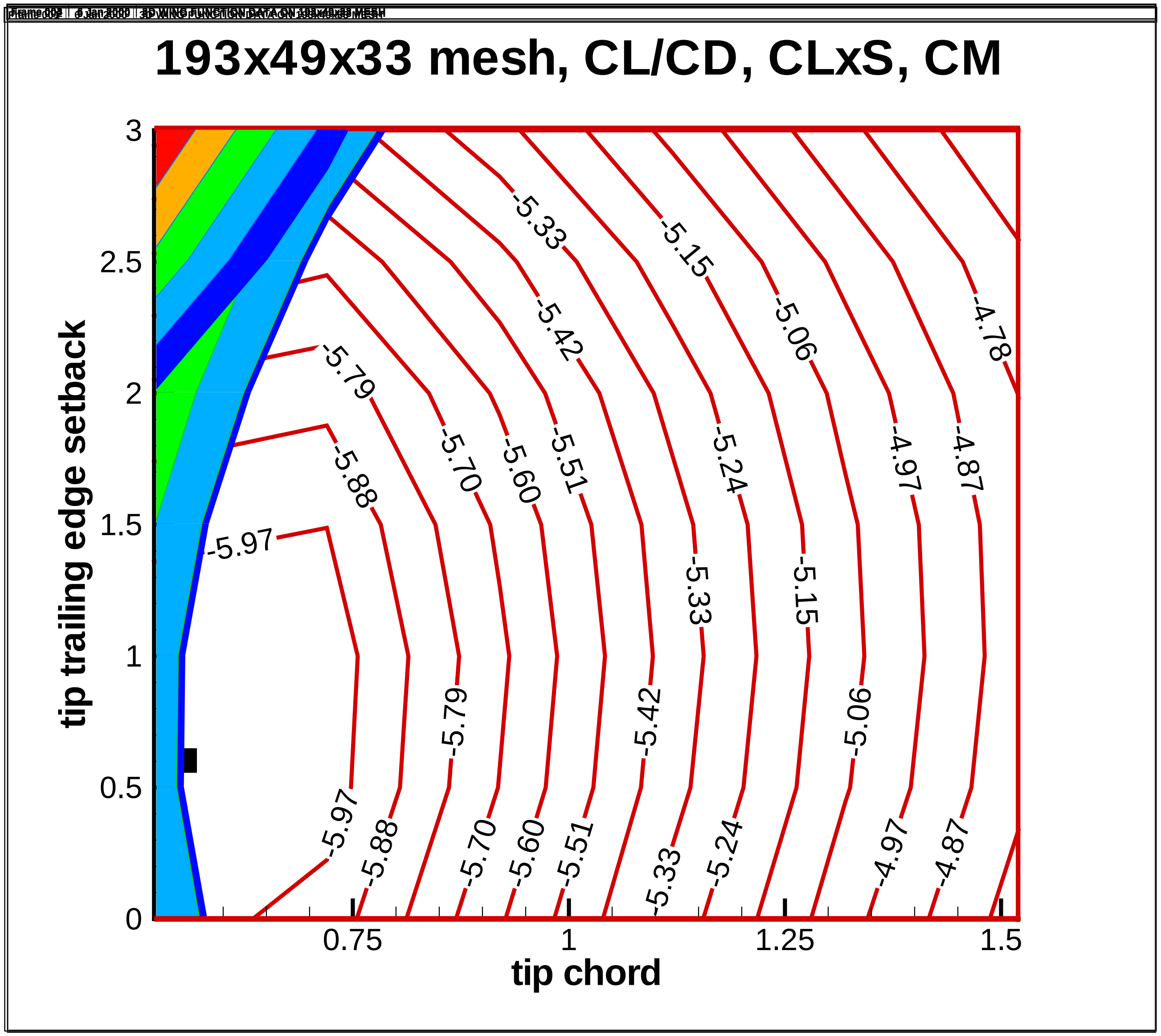
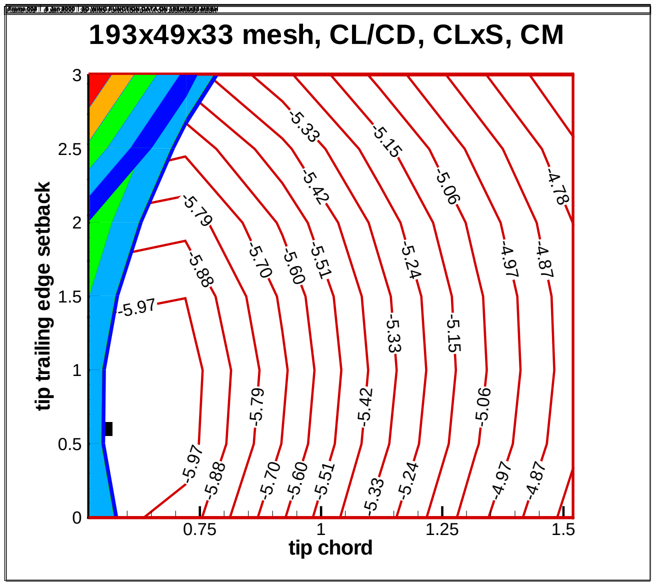
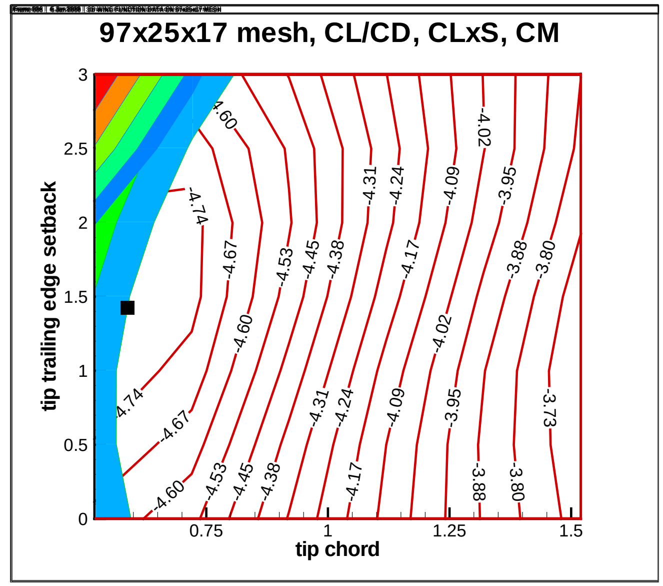
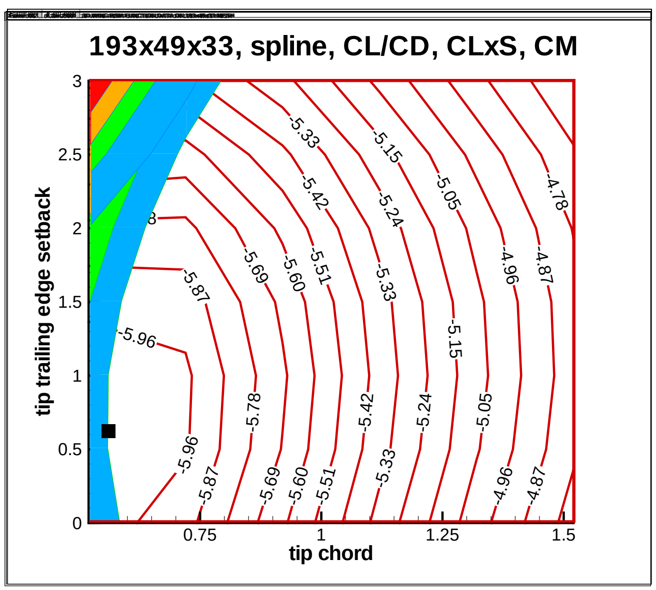
3D Wing Optimization: Problem Level Sets, Example

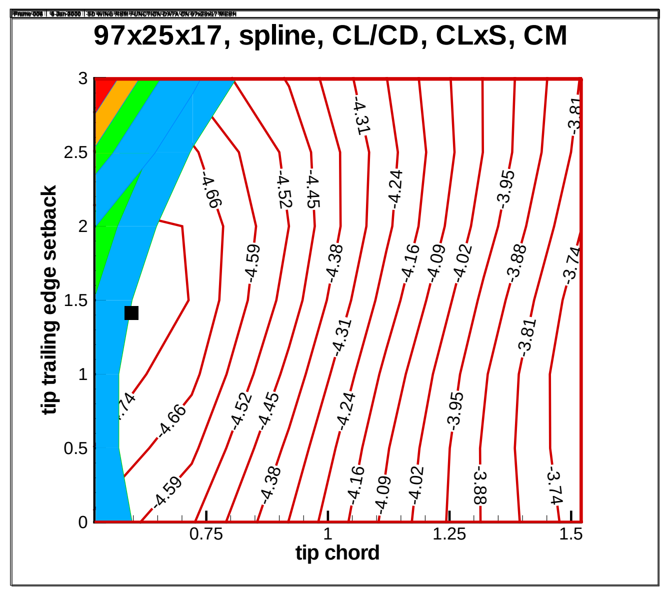


3D Wing Optimization: Actual Functions vs. Spline Substitutes









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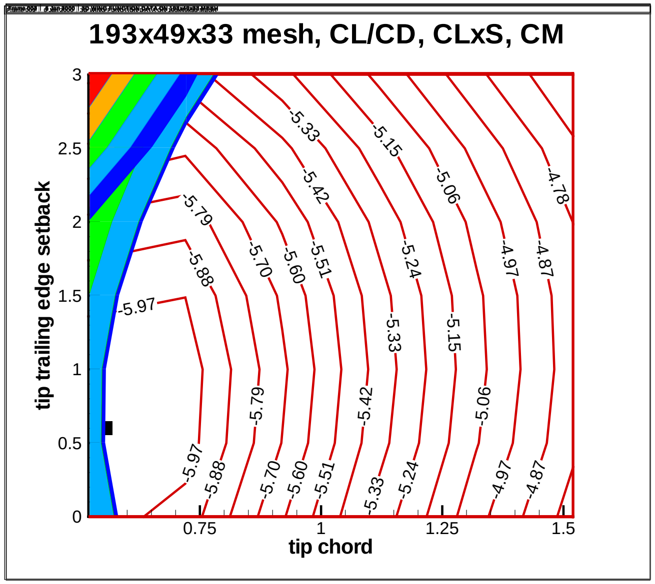
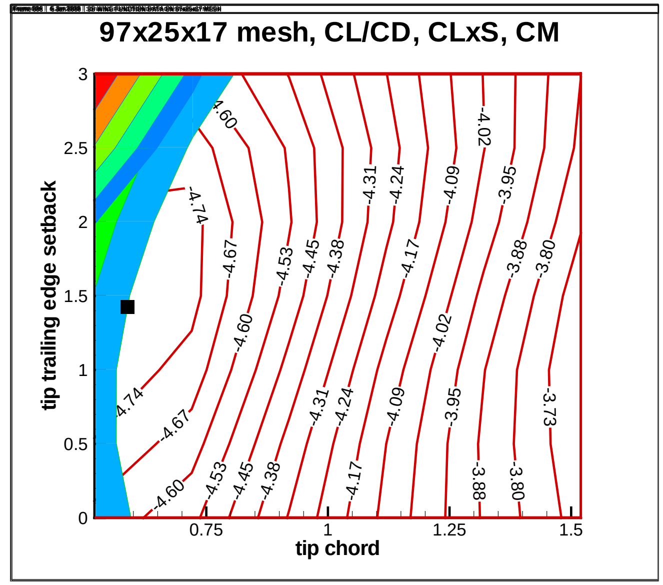
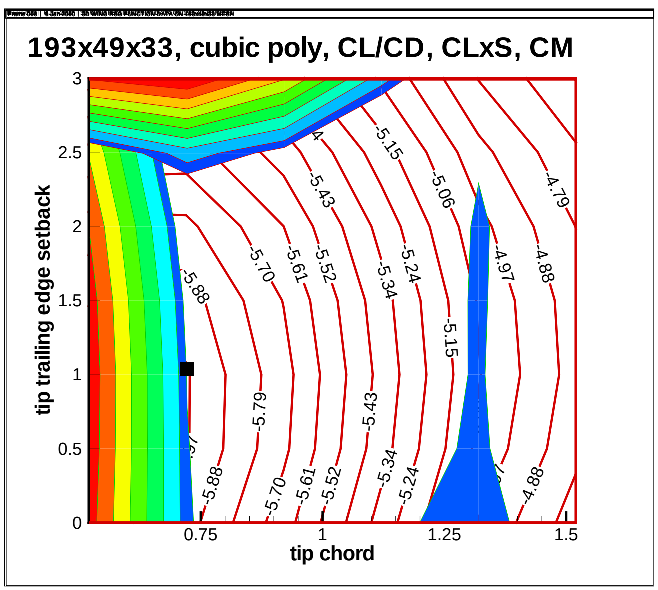
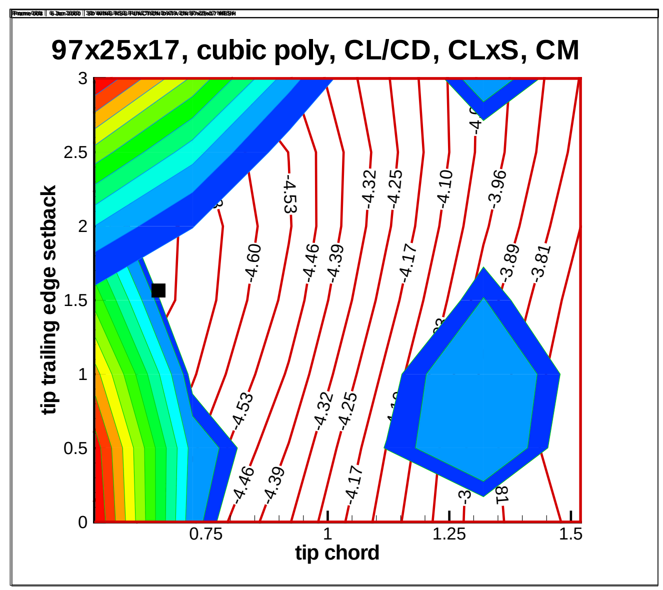
3D Wing Optimization: Actual Functions vs. Cubic Polynomial Substitutes









3D Wing Optimization: Discussion of Results

- Function evaluations, conventional SQP vs. SQP-AMF (number of sensitivity evaluations - same):

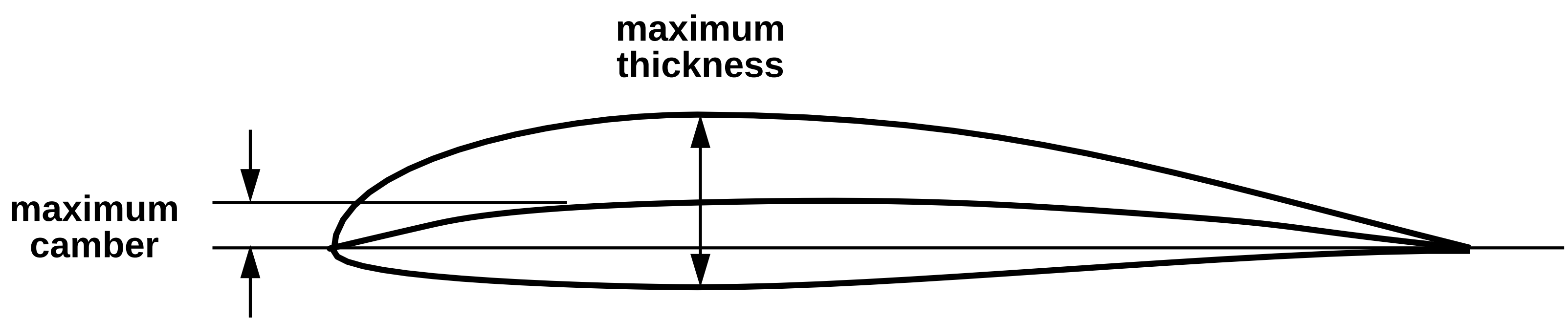
	hi-fi eval	lo-fi eval	equiv hi-fi eval	factor
Conventional SQP on poly	31		31	
SQP-AMF on poly	4	51	4 + 51/8 = 10 3/8	2.99
Conventional SQP on splines	21		21	
SQP-AMF on splines	4	28	4 + 28/8 = 7 1/2	2.8

- Optimization convergence criterion: 10^{-5}
- Optimization was done on RSM substitutes
- Savings across methods similar

2D Airfoil Optimization: Problem Description

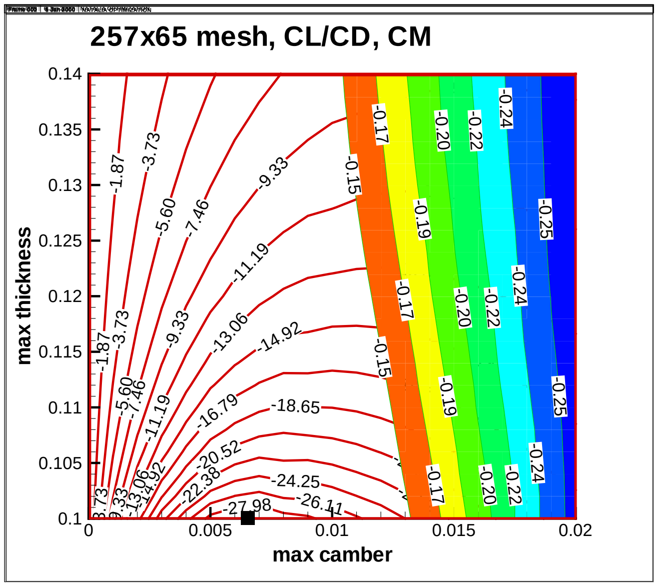
Problem formulated and assembled by L.L. Green

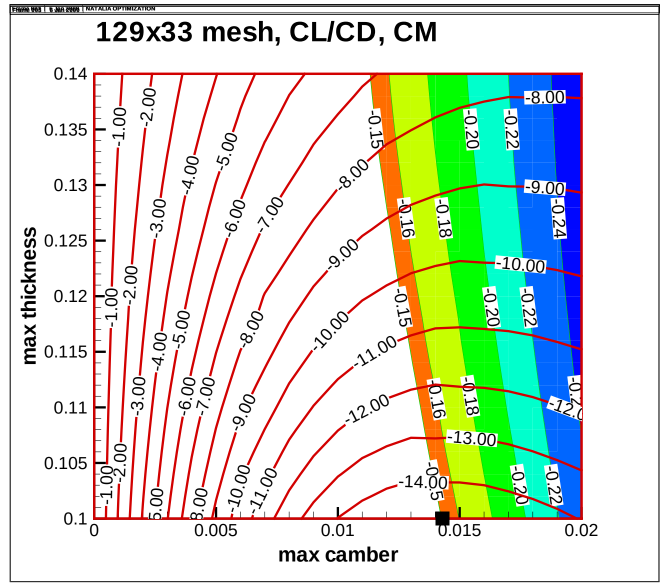
- **Analysis: Euler (NS/Euler code FLOMG, Swanson, Turkel)**
- **Design variables:**



- **Objective:** $-\frac{L}{D}$
- **Constraints: pitching moment**
- **Levels of fidelity: analyses on 257x65 and 129x33 meshes**
- **Time/analysis on 257x65 mesh = 4 Time/analysis on 129x33 mesh**
- **Approximately 8 min vs 2 min per analysis on SGI Octane**

2D Airfoil Optimization: Discussion of Results





- **Savings in function/sensitivity evaluations approximately twofold (factor ranging from 2.2 to 3.1) across all methods**
- **Savings lower than for the 3D wing problem due to lower computational expense**

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Conclusions to-Date

- **AMF has yielded threefold improvement for the 3D wing problem and twofold improvement for the airfoil problem, compared to non-AMF optimization**
- **Efficiency can be significantly improved if terminate inner-level problems as soon as sufficient decrease in the merit function is attained**
- **Variable-resolution models must use families of meshes**
- **Enforcing model consistency via beta-scaling works very well**
- **For single-discipline optimization and for the conventional formulation of multidisciplinary optimization, SQP-AMF appears to be the best choice**

Ongoing Work

- **AMF refinements:**
  - **Incorporating strategies for optimizing the use of lower-fidelity models (e.g., using information from *a posteriori* bounds for PDE outputs)**
- **Demonstrations on increasingly realistic problems:**
  - **3D wing problem in transonic regime**
  - **Larger number of design variables**
  - **Direct computations with analysis codes**
- **Other modeling options:**
  - **Optimization of a multi-element airfoil with variable-fidelity physics models (Euler vs. Navier-Stokes)**
  - **Direct use of RSM as low-fidelity models with systematic management**
- **Demonstrations for multidisciplinary problems**