On Multilevel Optimization Algorithms for Engineering Design Problems

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Outline

- The Problem Statement
- Example - Motivation - HSCT
- The “Perfect” Formulation
- Heuristic vs. Rigorous Formulations
- A Multilevel Approach to Equality Constrained Optimization
- Extension to MDO
- Computational Evaluation
  - Aerospike Engine Design
  - Method Evaluation Project
  - MDO Test Suite
- Summary

The Multidisciplinary (Design) Optimization (MDO) Problem

$$\text{minimize } f(x, u(x))$$

$$\text{subject to} \quad h(x, u(x)) = 0$$

$$g(x, u(x)) \geq 0$$

Where $u(x)$ is computed by solving the system $C(x, u(x)) = 0$ with

$$C(x, u(x)) = \begin{bmatrix} C_1(x, u_1(x), \ldots, u_M(x)) \\ \vdots \\ C_M(x, u_1(x), \ldots, u_M(x)) \end{bmatrix}$$

- **x - design variables, u - state variables**
- **$C(x, u(x))$ - system of PDE or ODE - multidisciplinary analysis**
- **Natural block structure; blocks - state equations for coupled disciplines or analyses (e. g., aerodynamics, structures, controls, propulsion, cost, etc.)**
- **Problem is multiobjective**

Example - Motivation

High Speed Civil Transport (HSCT)

- **Objective (HPCCP)**

  Demonstrate TERAFLOP computing on a model MDO problem using heterogeneous computing network

- **Consider**

  A representative set of disciplines, design criteria, design variables, etc.

HSCT Baseline Description

- Insert the postscript file with the description here.

System Versions

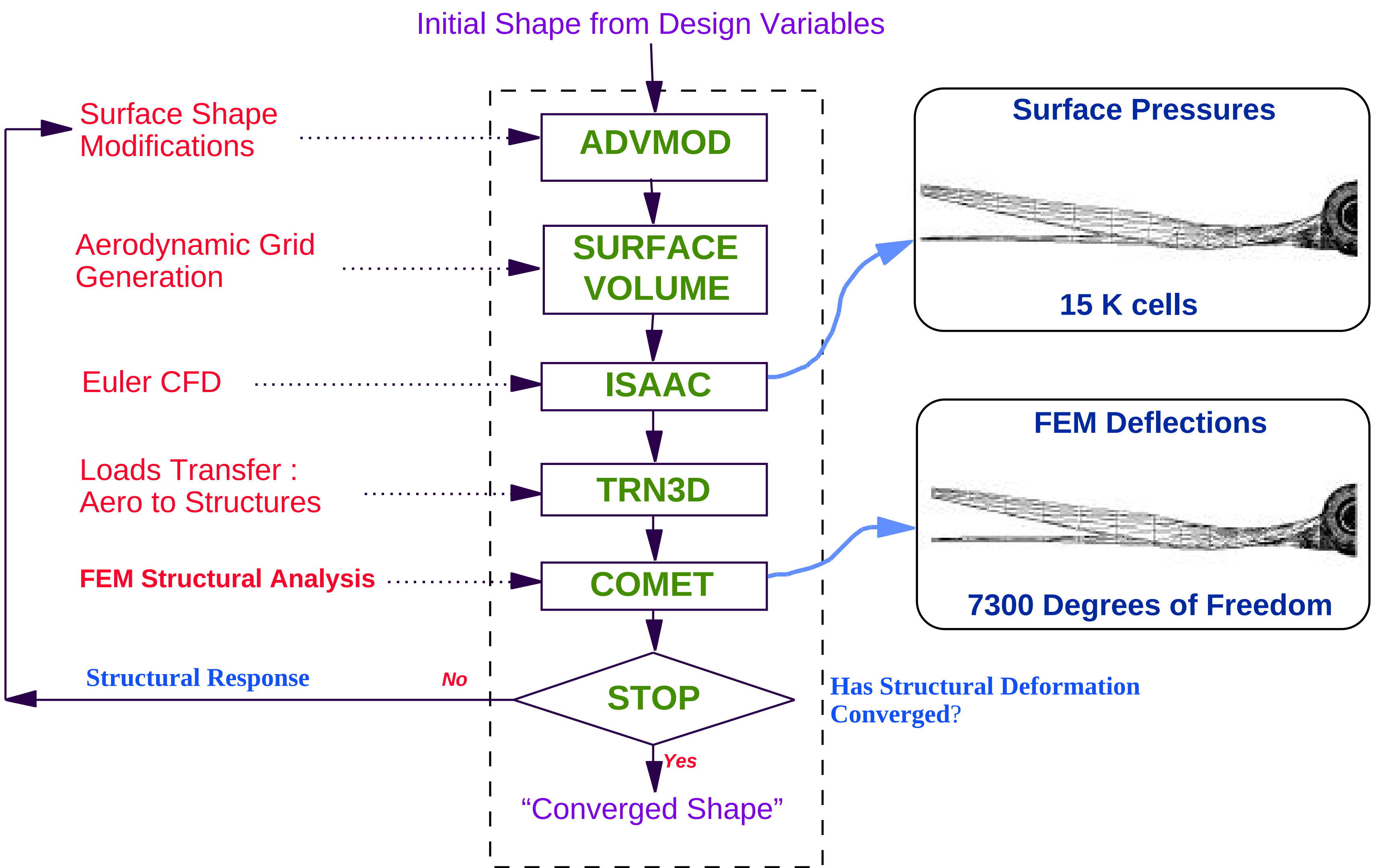
- Low Fidelity
  - Aerodynamic panel code
  - Equivalent-plate structures code
- Medium Fidelity
  - Euler CFD code
  - FEM structural code
  - Axisymmetric propulsion code

One aeroelastic function evaluation takes ≈ 6 hours on a heterogeneous network of 4-5 machines or ≈ 20 hours on a dedicated machine

- High Fidelity
  - N-S CFD code
  - Adaptive FEM model
  - 3-D propulsion code

One aeroelastic function evaluation is expected to require 5-6 days on a dedicated machine, 2 days on a parallel one, 3-6 hours on a 64-processor machine (O(102) hours total)

Novel Applications: HSCT - Key Steps



The Sources of Expense

- **Analyses and simulations are expensive both intrinsically and as a part of optimization process**
  - **Addressed by research on approximations in engineering optimization (Monday talk in the session Novel Applications II)**

- **Multidisciplinary Analysis (MDA) is expensive**
  - **In the HSCT example, the medium-fidelity case requires approximately 5 Gauss-Seidel iterations**
  - **Attempt to "break" the MDA loop**
  - **This and the sheer size and complexity of the MDO problem are addressed by research on MDO methods or "formulations"**

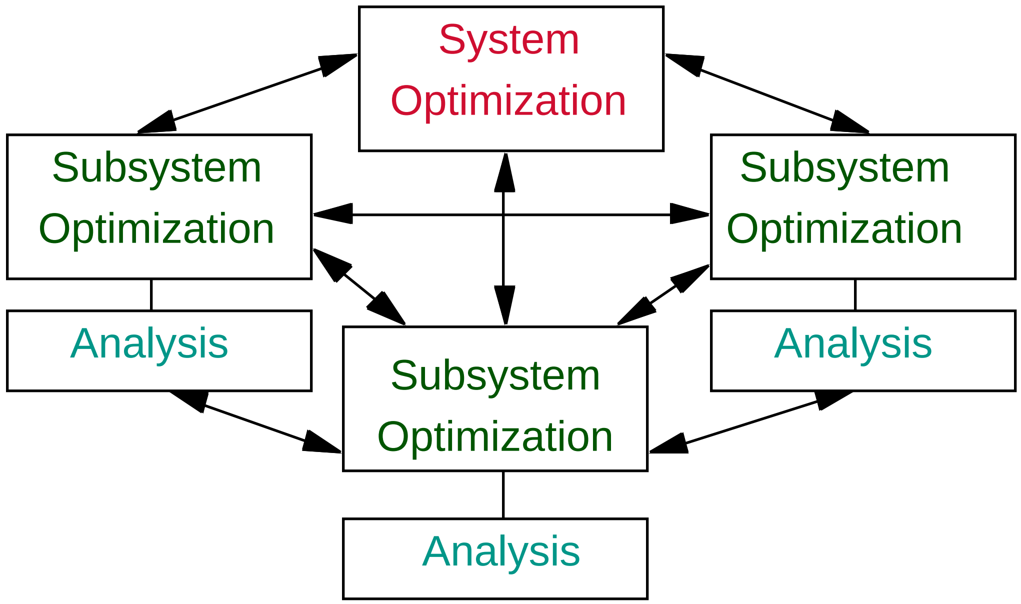
The “Perfect” Problem Formulation

- Efficient
- Autonomous, parallel / distributed processing of components
- Convergence / robustness
- Arbitrary strength and bandwidth of coupling
- Exploit full or partial separability
- Interactive (designer-in-the-loop) vs. automatic processing
- Flexible and varied optimization techniques
- Arbitrary number of variables and constraints
- Multiobjective capabilities
- Models and approximations of varying fidelity
- Ease of use / decomposition / coordination / implementation
- Correct answers

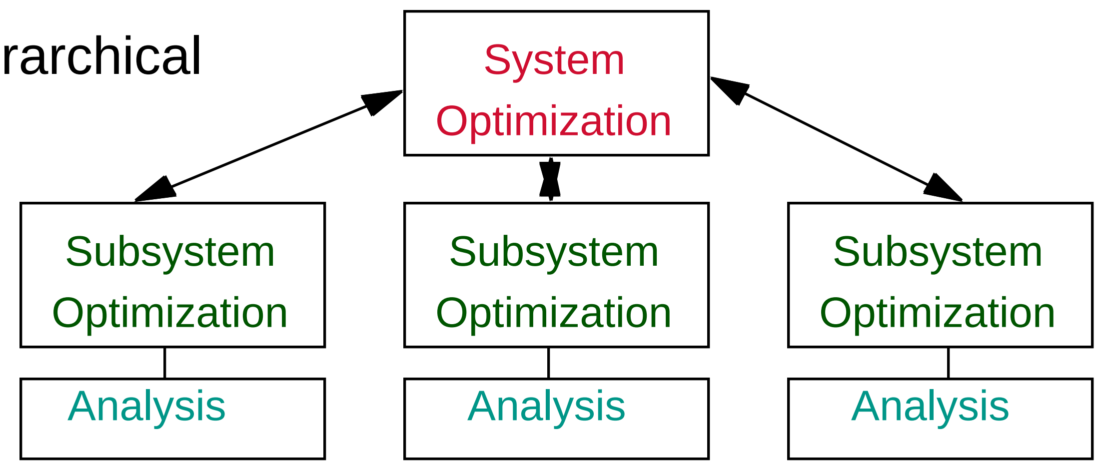
Heuristic vs. Rigorous Problem Formulations

Some Heuristic Formulations

- Non-hierarchical



- Hierarchical



Collaborative Optimization: an Example of a Heuristic Formulation
(Schoeffler 1971, Braun et al. 1994)

The Original Problem:

minimize $J(x)$

subject to $C(x) \geq 0$

where $C(x) = \begin{bmatrix} C_1(x) \\ \dots \\ C_N(x) \end{bmatrix}$

Partition x into $\{x_1, \dots , x_N\}$, not necessarily disjoint.

System-Level Coordination

minimize $J_{syst}(z)$

subject to $g^*_j(z) = 0, j=1, \dots , N$

z_1 g^*_1 z_N g^*_N

Subproblem 1

$\min g_1(x_1) = \Sigma (x_{i1} - z_{i1})^2 + \Sigma (y_{i1} - z_{i1})^2$

subject to $C_1(x_1, \xi_1) \geq 0$

. . .

Subproblem N

$\min g_N(x_N) = \Sigma (x_{iN} - z_{iN})^2 + \Sigma (y_{iN} - z_{iN})^2$

subject to $C_N(x_N, \xi_N) \geq 0$

x_i, ξ_i are inputs to analysis i; y_i, C_i, g_i are its outputs.

Collaborative Optimization (cont.)

+ Parallel, autonomous processing of disciplines
+ Consistent with design environment
+ Consistent with organizational structures

- Not robust
- Convergence properties in question

Δ Still undergoing changes
Δ Promising

Some Rigorous Formulations

(Schoeffler 1971, Cramer et al. 1993, Lewis 1997)

- **The Original Problem**

$$\min f(x, u(x))$$

where, given x, u(x) is computed by solving the system

$$C(x, u(x)) = \begin{bmatrix} C_1(x, u_1(x), \ldots , u_M(x)) \\ \cdots \\ C_M(x, u_1(x), \ldots , C_M(x)) \end{bmatrix}$$

- **The Conventional Approach** **(Variable Reduction, "Multidisciplinary Feasible" (MDF), All-in-One)**

$$\min_{x} f(x, u_1(x), \ldots , u_M(x))$$

where, given the input design variables x, MDA is computed at each optimization iteration:

$$C_i(x, u_1(x), \ldots , u_M(x)) = 0, \; i = 1, \ldots , M$$

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Some Rigorous Formulations (Cont.)

- **The Nonlinear Programming Approach** ("All-at-Once" (AAO), "Simultaneous Analysis and Design" (SAD or SAND), "No-Discipline Feasible (NDF))

$$\underset{x,\, u_1,\, \ldots\,,\, u_M}{\text{minimize}}\ f(x, u_1, \ldots , u_M)$$

$$\text{subject to } C_1(x, u_1, \ldots , u_M) = 0$$

$$\cdots$$

$$C_M(x, u_1, \ldots , u_M) = 0$$

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- **The In-Between Approach** ("Individual Discipline Feasible" (IDF), "Some-Discipline Feasible" (SDF))

E.g., $$\underset{x,\, u_1,\, \ldots\,,\, u_{i-1},\, u_{i+1},\, \ldots,u_M}{\text{minimize}}\ f(x, u_1, \ldots , u_i(x,\ldots,u_{i-1},u_{i+1},\ldots,u_M), \ldots,u_M)$$

$$\text{subject to } C_1(x, u_1, \ldots , u_i(x,\ldots,u_{i-1},u_{i+1},\ldots,u_M), \ldots,u_M) = 0$$

$$\cdots$$

$$C_M(x, u_1, \ldots , u_i(x,\ldots,u_{i-1},u_{i+1},\ldots,u_M), \ldots,u_M) = 0$$

and u_i is computed by solving

$$C_i(x, u_1, \ldots , u_i(x,\ldots,u_{i-1},u_{i+1},\ldots,u_M), \ldots,u_M) = 0$$

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A Multilevel Approach
(Alexandrov 1993, et al. 1997)

- **Assuming the IDF approach, how does one solve a large block - structured, fully-coupled or arbitrarily-coupled problem?**

- **The proposed method is a trust-region, block, null-space approach to solving large-scale NLP.**

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The Multilevel Algorithm for Equality Constrained Optimization

Given $x_c \quad R^n$, $\delta^c_k > 0$, $k = 1, \ldots, M+1$, **and other trust-region parameters,**

do until convergence

$y_0 = x_c$

do $k = 1, M$ **(block-linearized feasibility)**

Compute approximate solution s_k to

minimize $\|C_k(y_{k-1}) + \quad C_k^T(y_{k-1})\, s\|^2$

subject to $\quad C_j^T(y_{j-1})\, s = 0,\ j = 1, \ldots, k-1$

$\|s\| \leq \delta^c_k$

$y_k = y_{k-1} + s_k$

end do

Compute (optimality step) approximation solution s_{M+1} to

minimize $f(y_M) + \quad f^T(y_{M+1})\, s + 1/2\, s^T H_M s$

subject to $\quad C_j^T(y_{j-1})\, s = 0,\ j = 1, \ldots, k-1$

$\|s\| \leq \delta^c_k$

$y_{M+1} = y_M + s_{M+1}$

$s_c = \sum_{k=1}^{M+1} s_k$

Update the penalty parameters, x_c, δ^c_k, $k = 1, \ldots, M$

end do

Computing the Steps

- Each substep must satisfy a sufficient decrease condition (Fraction of Cauchy Decrease) on the subproblem it solves.
- General block-linearly feasible steps (reduced basis steps) are especially appropriate for MDO:

1. Partition $C_1^T(y_0) = [B_1 \mid N_1]$, where B_1 is an invertible matrix possibly obtained by column permutations. Then computing

$$s_1^{lin} = [-B_1^{-1} C_1(y_0), 0]^T$$

we have

$$s_1 = \frac{\delta^c{}_1 \, s_1^{lin}}{\|s_1^{lin}\|} \qquad \text{and } y_1 = y_0 + s_1$$

2. Partition $C_2^T(y_1)\, Z_1 = [B_2 \mid N_2]$,

where $$Z_1 = \begin{bmatrix} -B_1^{-1} N_1 \\ I \end{bmatrix}$$

and continue.

Measuring Progress

- **The Merit Function**

$$P(x; \rho_1, \dots, \rho_M) \equiv f(x) + \sum_{k=1}^{M} \left(\prod_{j=k}^{M} \rho_j\right) \|C_k(x)\|^2$$

E.g., for M=2,

$$P(x; \rho_1, \rho_2) \equiv f(x) + \rho_2\left(\|C_2(x)\|^2 + \rho_1 \|C_1(x)\|^2\right)$$

The augmented lagrangian can be used (and should be, for performance)

- **The Model**

$$M(s_c; \rho_1, \dots, \rho_M) \equiv \varphi(s_{M+1}) + \sum_{k=1}^{M} \left(\prod_{j=k}^{M} \rho_j\right) \|C_k(y_{k-1}) + C^T_k(y_{k-1})s_k\|^2$$

quadratic model of the objective or the lagrangian

Other Features

- **Updating the Penalty Parameters**
  - Extension of El-Alem scheme (1988, 1991)
- **Updating the Trust-Region Radii**
  - Analysis accommodates a number of techniques (Alexandrov 1997)
- **Convergence Analysis**
  - Under "standard" assumptions, at least a subsequence of the generated sequence of iterates converges to a stationary point of the problem
- **Numerical Testing**
  - On Hock & Schittkowski test set - comparable to NPSOL, KSOPT, CONMIN

Extension to MDO

- Include arbitrary first order approximations (Alexandrov 1997)
- Extension to general nonlinear programming - equality and inequality constraints (in progress, Alexandrov, El-Alem)
- The Problem:

$$\begin{aligned} &\min f(x) \\ &\text{s.t. } h(x) = 0 \\ &\qquad g(x) \leq 0 \end{aligned}$$

$f : R^n \to R$, $h : R^n \to R^m$, $g : R^n \to R^p$, $m < n$, at least twice continuously differentiable.

Sketch of the Algorithm

Given x_k R^n, λ_k, μ_k, A_k (indicators), Δ_k, and other trust-region parameters,

1. Compute $s^{CP,g}$ for g from x_k, using A_k

Compute W_k - an indicator matrix for $s^{CP,g}$

Compute approximate solution $s^g{}_k$ to

$$\min ||W_k (g_k + \quad g^T{}_k s^g)||^2$$

$$\text{s.t. } ||s^g|| \leq \tau_1 \Delta_k, \tau_1 \quad [0.5, 06]$$

2. Compute $Z^g{}_k \equiv$ basis for N $(W_k \quad g^T{}_k)$

Compute approximate solution $s^h{}_k$ to

$$\min ||h_k + (Z^g{}_k \quad h_k)^T s^h||^2$$

$$\text{s.t. } ||Z^g{}_k s_k|| \leq \tau_2 \Delta_k, \tau_2 \quad [0.6, 0.8]$$

3. Compute $Z^{gh}{}_k \equiv$ basis for $N \begin{bmatrix} & h_k & \\ W_k & & g^T{}_k \end{bmatrix}$

Compute approximate solution $s^t{}_k$ to

min model of the lagrangian reduced by $Z^{gh}{}_k$

$$\text{s.t. } ||Z^{gh}{}_k s^t|| \leq \sqrt{\ } \Delta^2{}_k - (||s_k{}^h||^2 + ||Z_k{}^g s_k{}^h||^2)$$

4. Set $s_k = s_k{}^g + Z^g{}_k s_k{}^h + Z^{gh}{}_k s_k{}^t$

. . .

Computational Evaluation

- **The Aerospike Engine Design Problem**
  - **A realistic MDO problem of practical interest**
    - **Develop and demonstrate MDO capabilities for SSTO engine concepts**
    - **Assess performance of various approaches to MDO**
    - **Several levels of fidelity are available**
  - **Problem Features:**
    - **Components:**
      - Aerodynamics, structures, trajectory, others
    - **Minimize GLOW (Gross Lift-Off Weight) subject to structural constraints**
    - **One case: 16 variables, 596 structural constraints**
    - **Multidisciplinary feasible formulation used as a base case**
    - **≈ 1 day to obtain solution (≈ 20 iterations) on a Sun ULTRA 1**

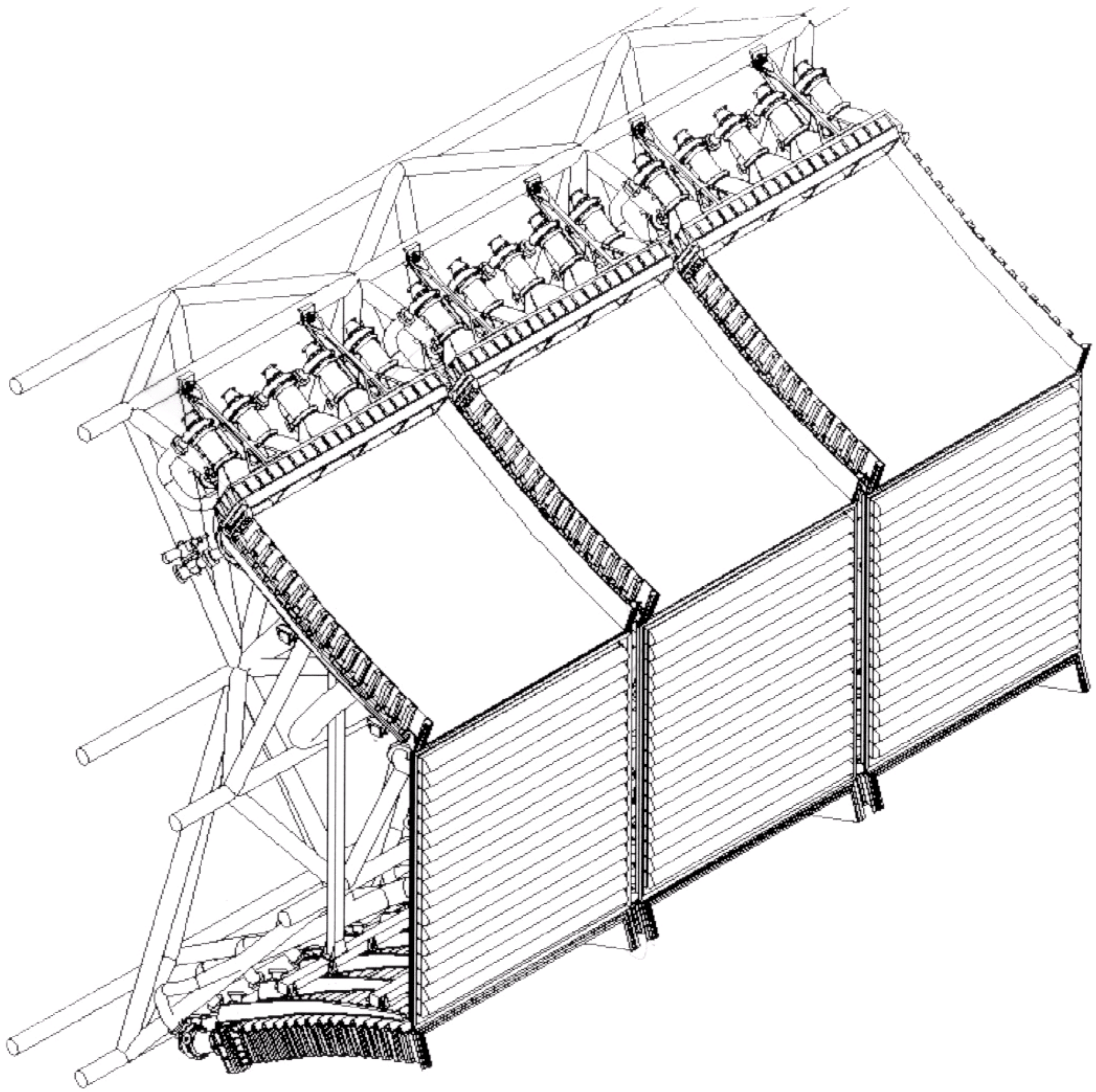
RLV X-33 Concepts

Lockheed Martin

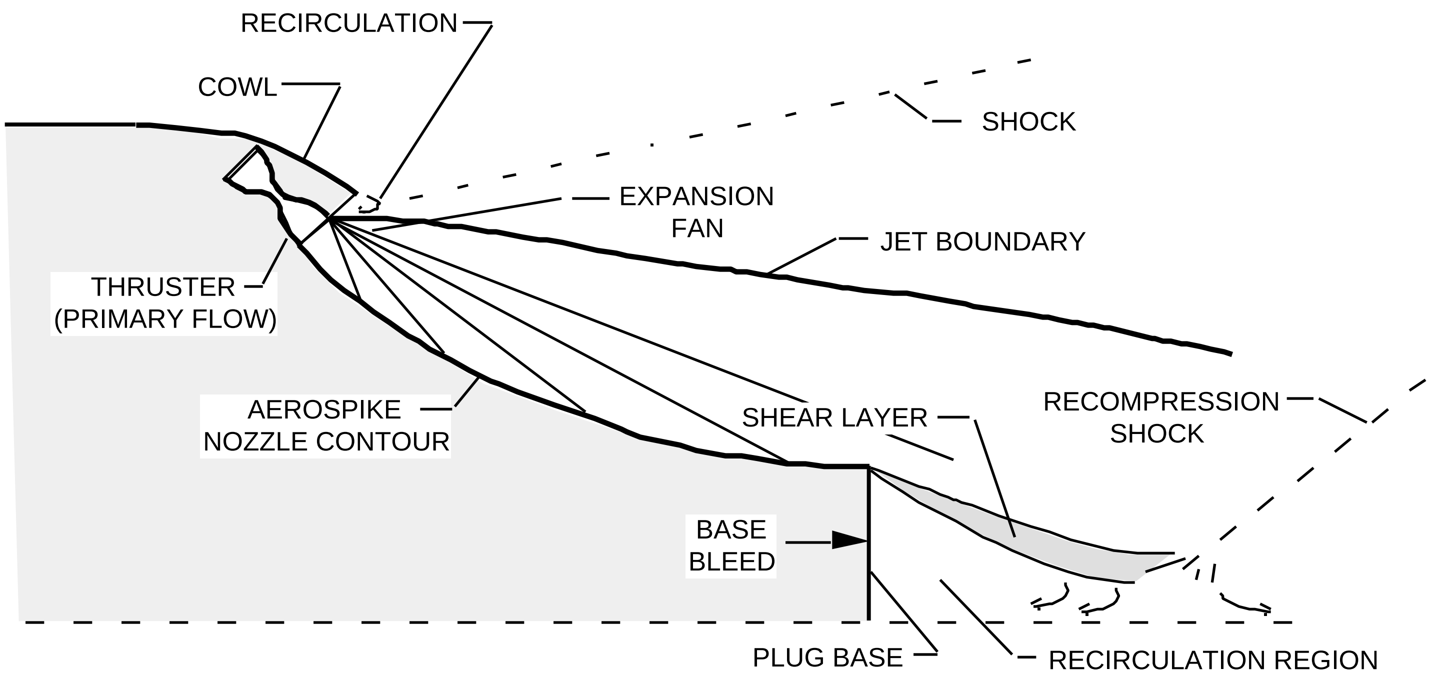
McDonnell Douglas/Boeing

Rockwell

AEROSPIKE ENGINE

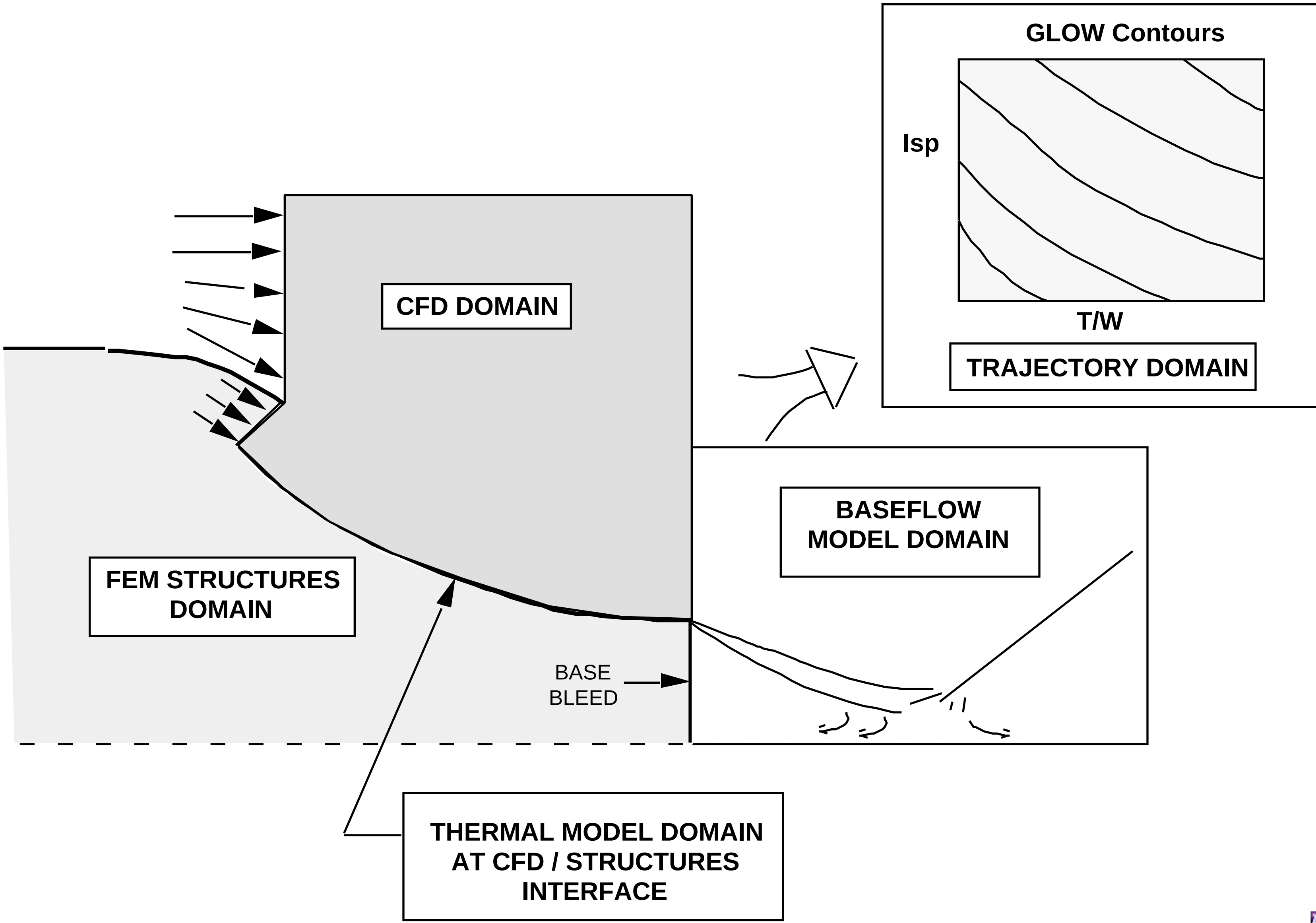


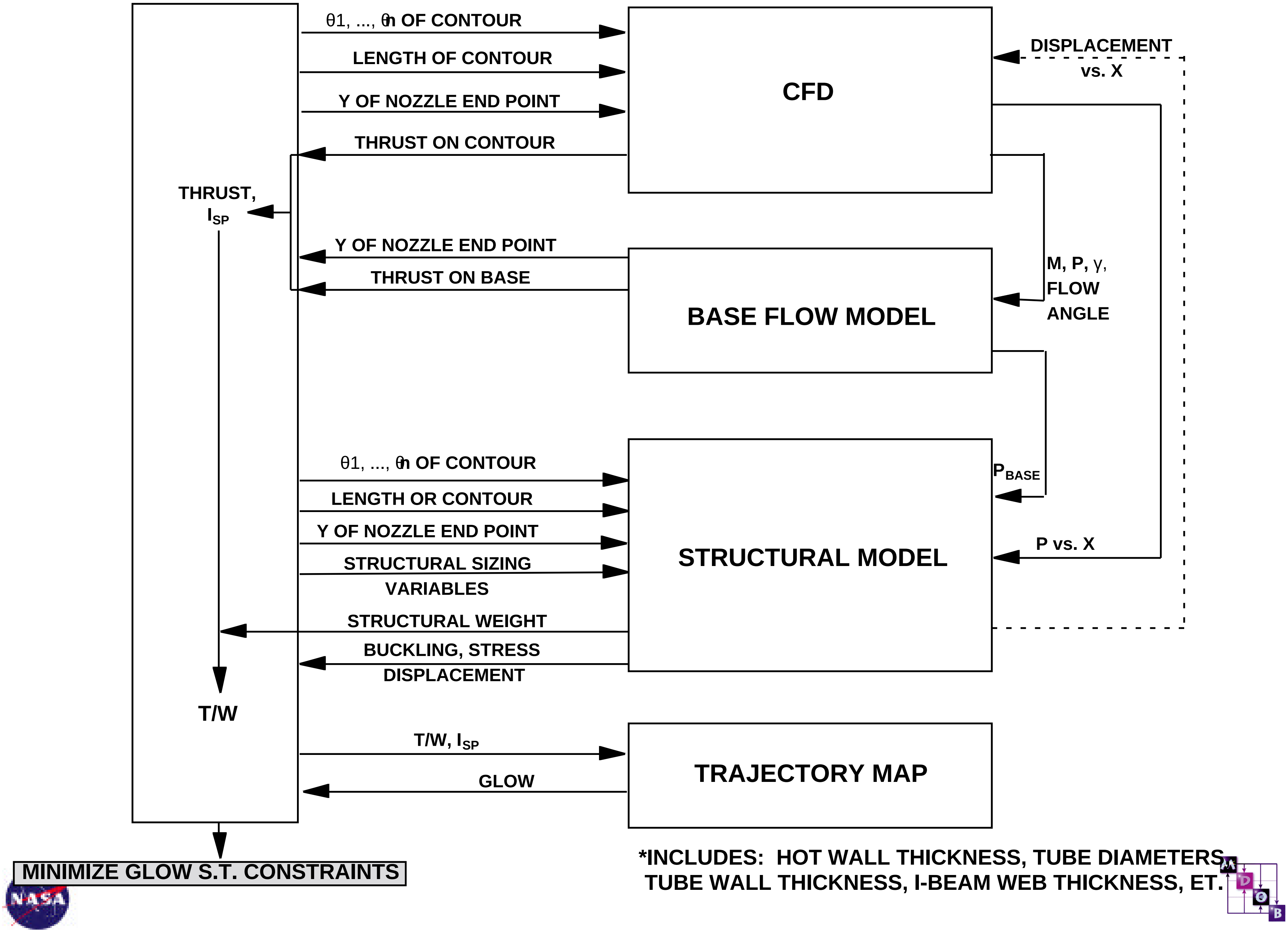
Aerospike Nozzle Flowfield Characteristics



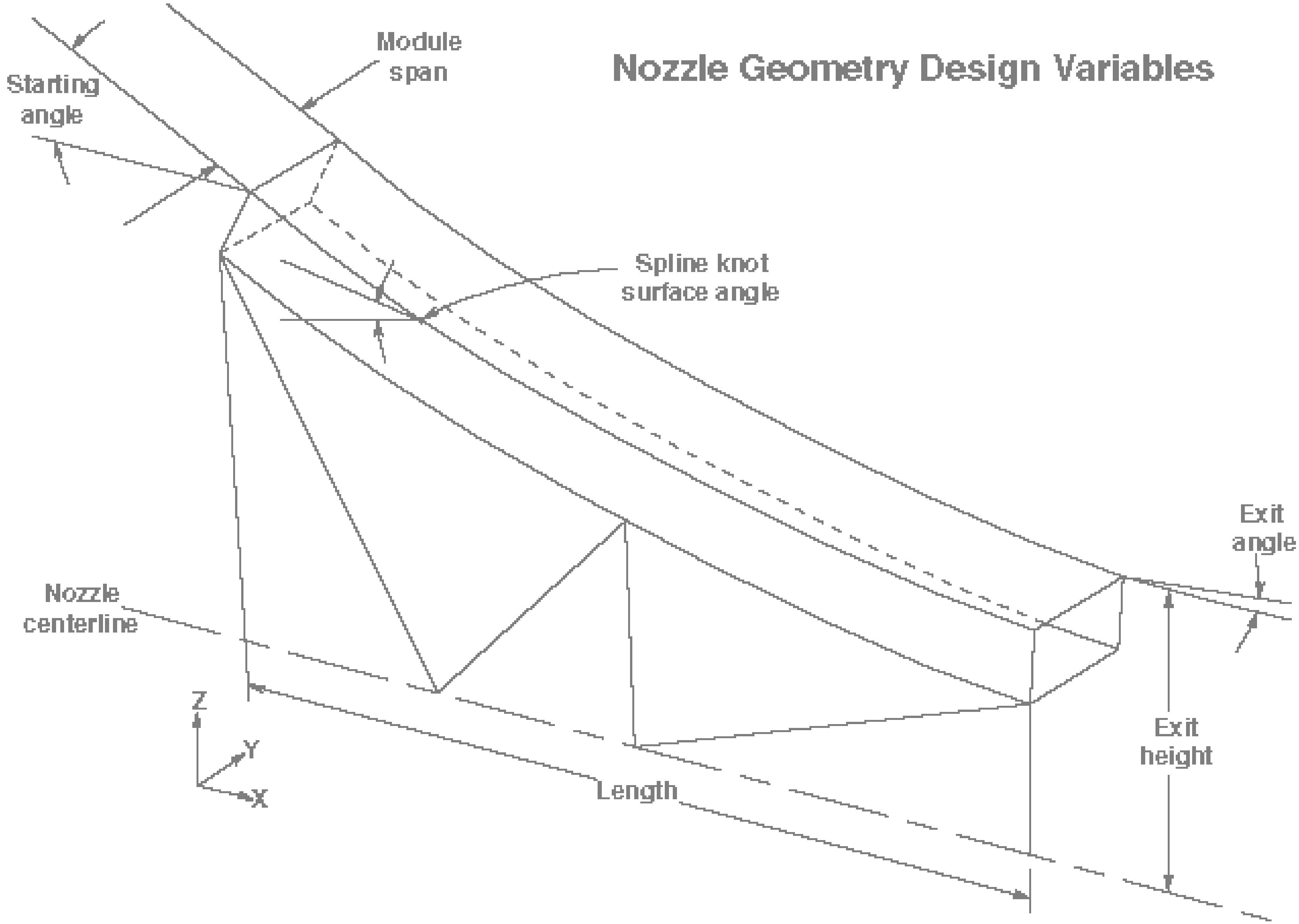
CFD-96-OPTIMIZATION DISK/WWF

AEROSPIKE MDO DOMAIN DECOMPOSITION

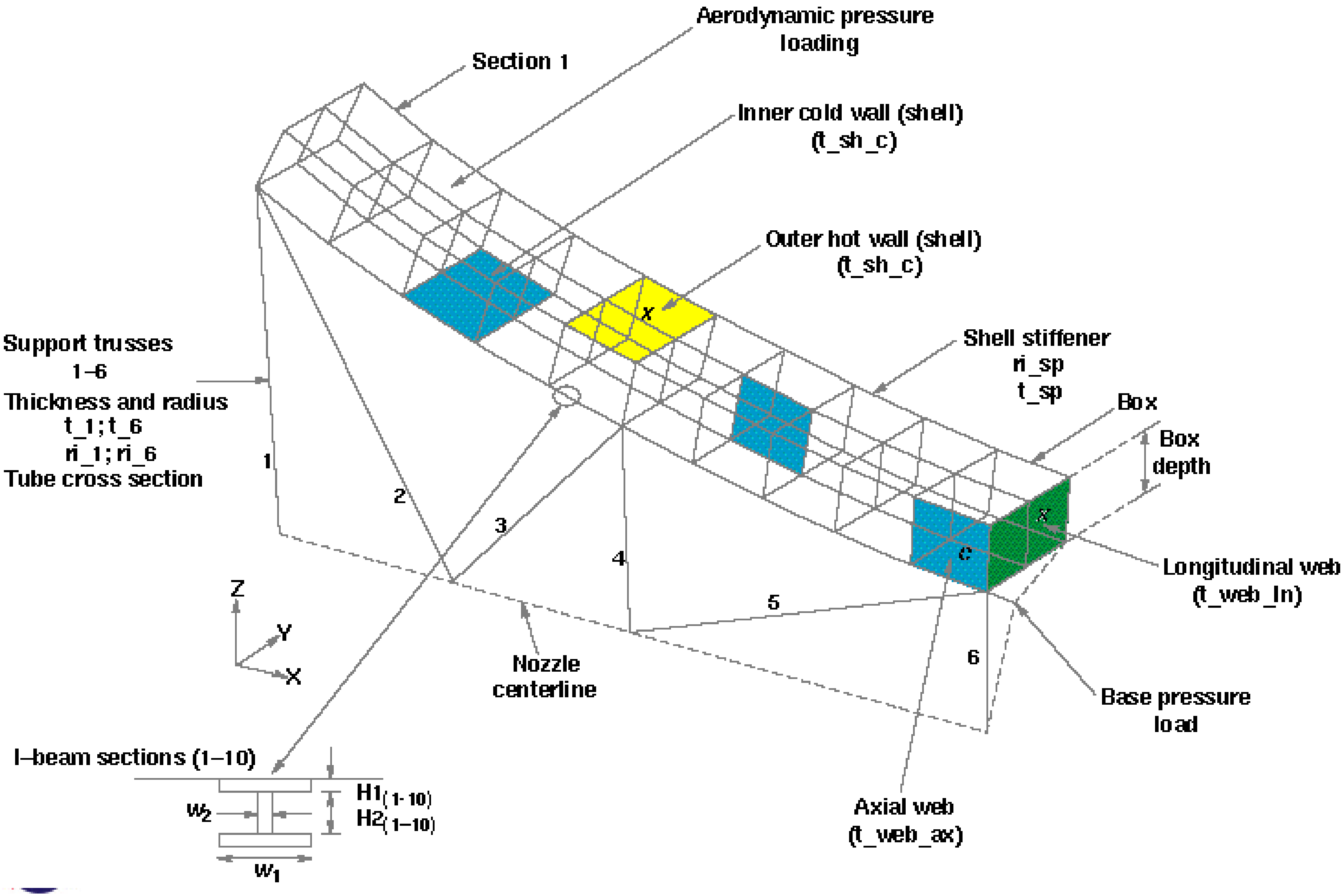



θ1, ..., θn OF CONTOUR
LENGTH OF CONTOUR
Y OF NOZZLE END POINT
THRUST ON CONTOUR
CFD
DISPLACEMENT
vs. X
THRUST,
I_{SP}
Y OF NOZZLE END POINT
THRUST ON BASE
BASE FLOW MODEL
M, P, γ,
FLOW
ANGLE
θ1, ..., θn OF CONTOUR
LENGTH OR CONTOUR
Y OF NOZZLE END POINT
STRUCTURAL SIZING
VARIABLES
STRUCTURAL WEIGHT
BUCKLING, STRESS
DISPLACEMENT
STRUCTURAL MODEL
P_{BASE}
P vs. X
T/W
T/W, I_{SP}
GLOW
TRAJECTORY MAP
MINIMIZE GLOW S.T. CONSTRAINTS
*INCLUDES: HOT WALL THICKNESS, TUBE DIAMETERS,
TUBE WALL THICKNESS, I-BEAM WEB THICKNESS, ET.

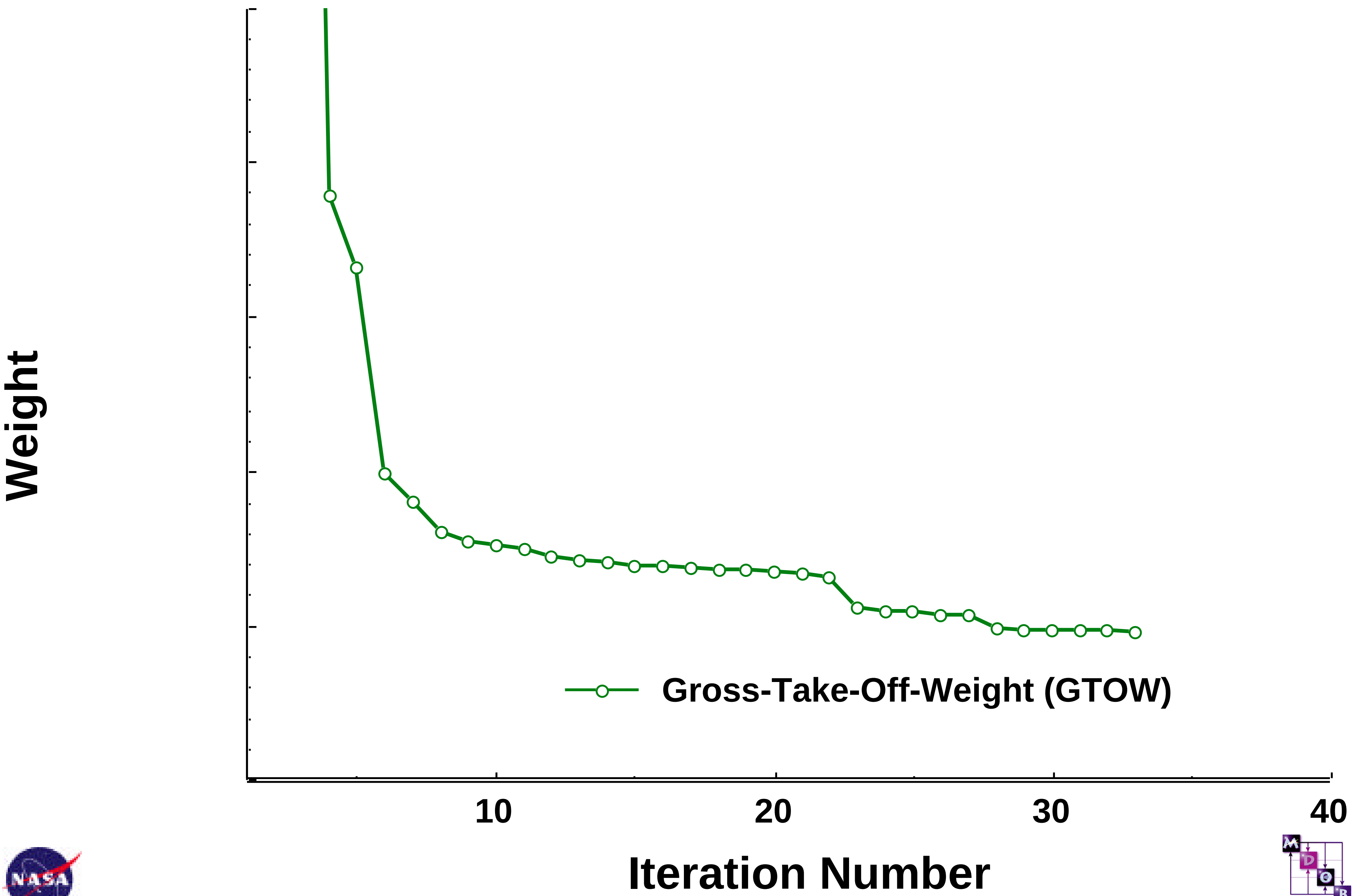
Nozzle Geometry Design Variables

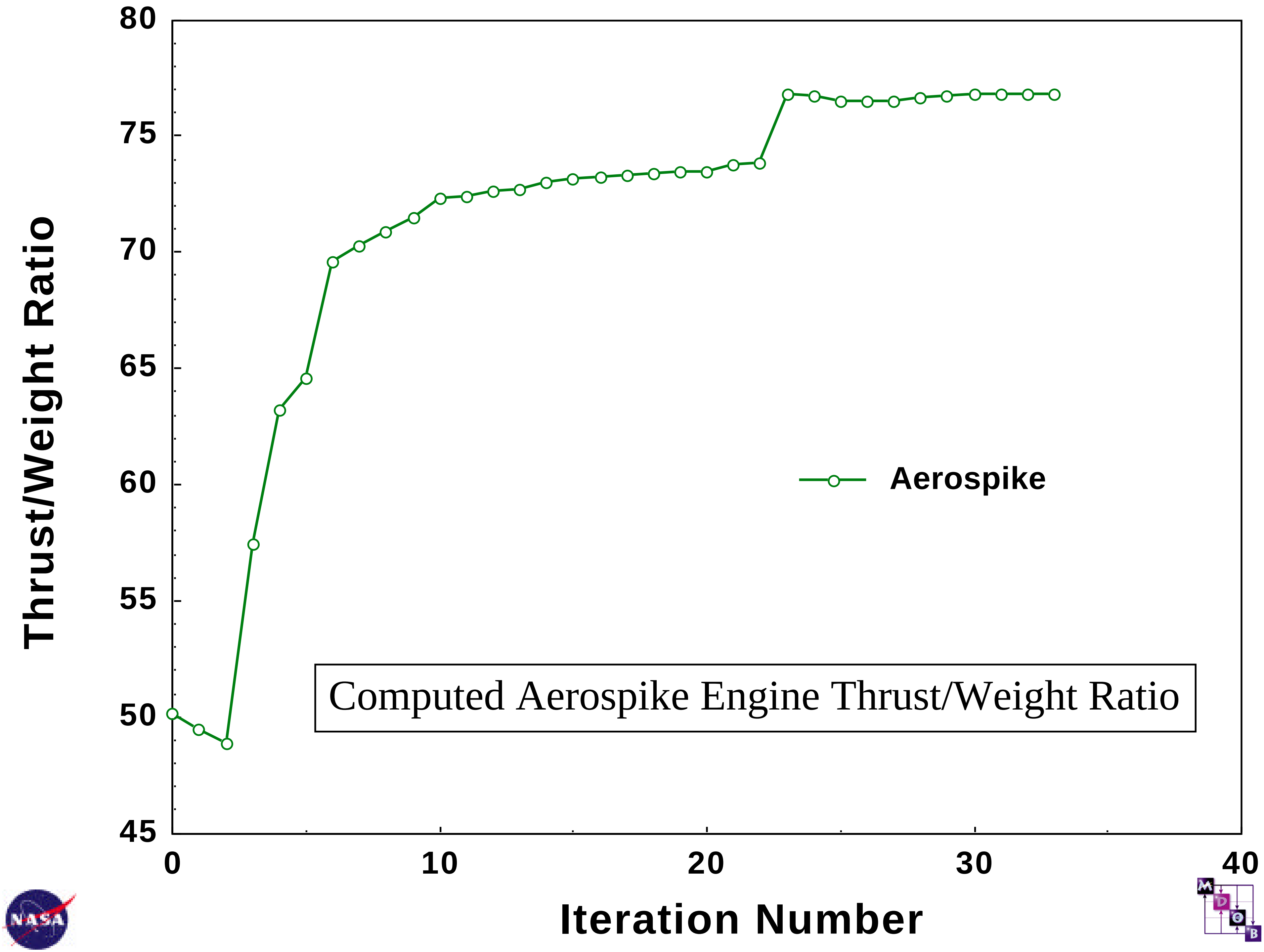


Structural Loading and Design Variables



Objective Function




80
75
70
65
60
55
50
45
0
10
20
30
40
Thrust/Weight Ratio
Iteration Number
Aerospike
Computed Aerospike Engine Thrust/Weight Ratio
NASA

Method Evaluation Project

- **Difficulty**
  - **Vary scarce computational evidence on basic algorithmic properties of MDO methods**
- **HPCCP office at NASA Langley funded a method evaluation project with MDOB and ESI, Inc.**
  - **Selected an initial set of methods and MDO problems, and a large set of performance characteristics**
  - **Preliminary report expected in October 1997**
- **MDO Test Suite**
  - **MDO problems arranged by degree of complexity**
  - **Access from MDOB homepage:**

    http://fmad-www.larc.nasa.gov/MDOB
  - **Comments and contributions welcome!**

Summary

- MDO is a complex NLP with special structure
- Much work is being done on methods for MDO; considerable part is based on heuristics
- Here described a number of analytically rigorous methods and one method for solving strongly or arbitrarily coupled problems

Current work

- Constraints
- Multiple objectives
- Integration of approximations
- MDO issues at various design levels (conceptual, preliminary, detailed)
- Method evaluation and classification
- Demonstration on realistic problems

- For reports and software write to: n.alexandrov@larc.nasa.gov