Challenges in simulation-based optimization

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This, of course, does not do justice to the real problem.

Efficiency in solving MDO problems

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- Efficiency in problem integration.
- Computational efficiency of disciplinary components.

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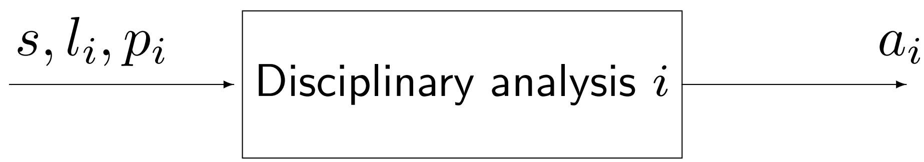
	1	2	3	4	5	6	7
FIO	610	220	610	81	3234	5024	8730
CO	15626	19872	1785	2102	837	40125	691058
DAO	9530	8796	382	-	544	932	-

Cost of optimization in terms of the number of analyses required,
3 different formulations, 7 test problems.
Alexandrov and Kodiyalam (1998)

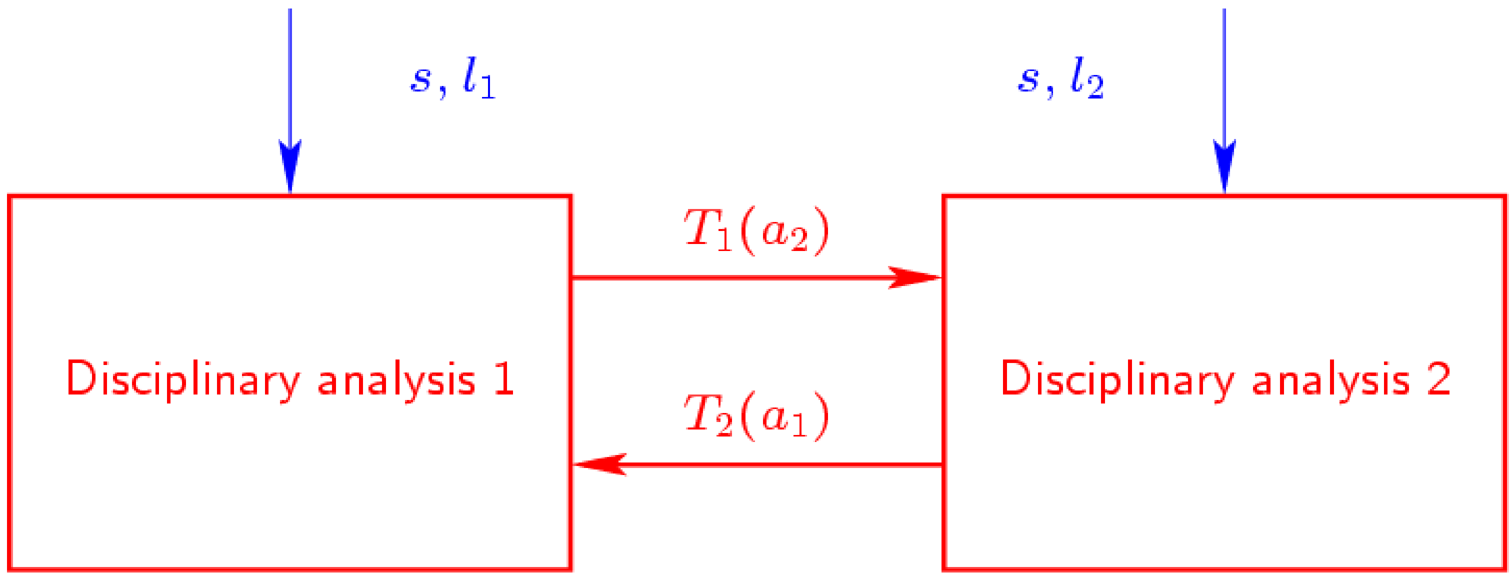
Disciplinary analysis



Disciplinary analysis:

$$A_i(a_i; p_i, s, l_i) \quad = \quad 0.$$

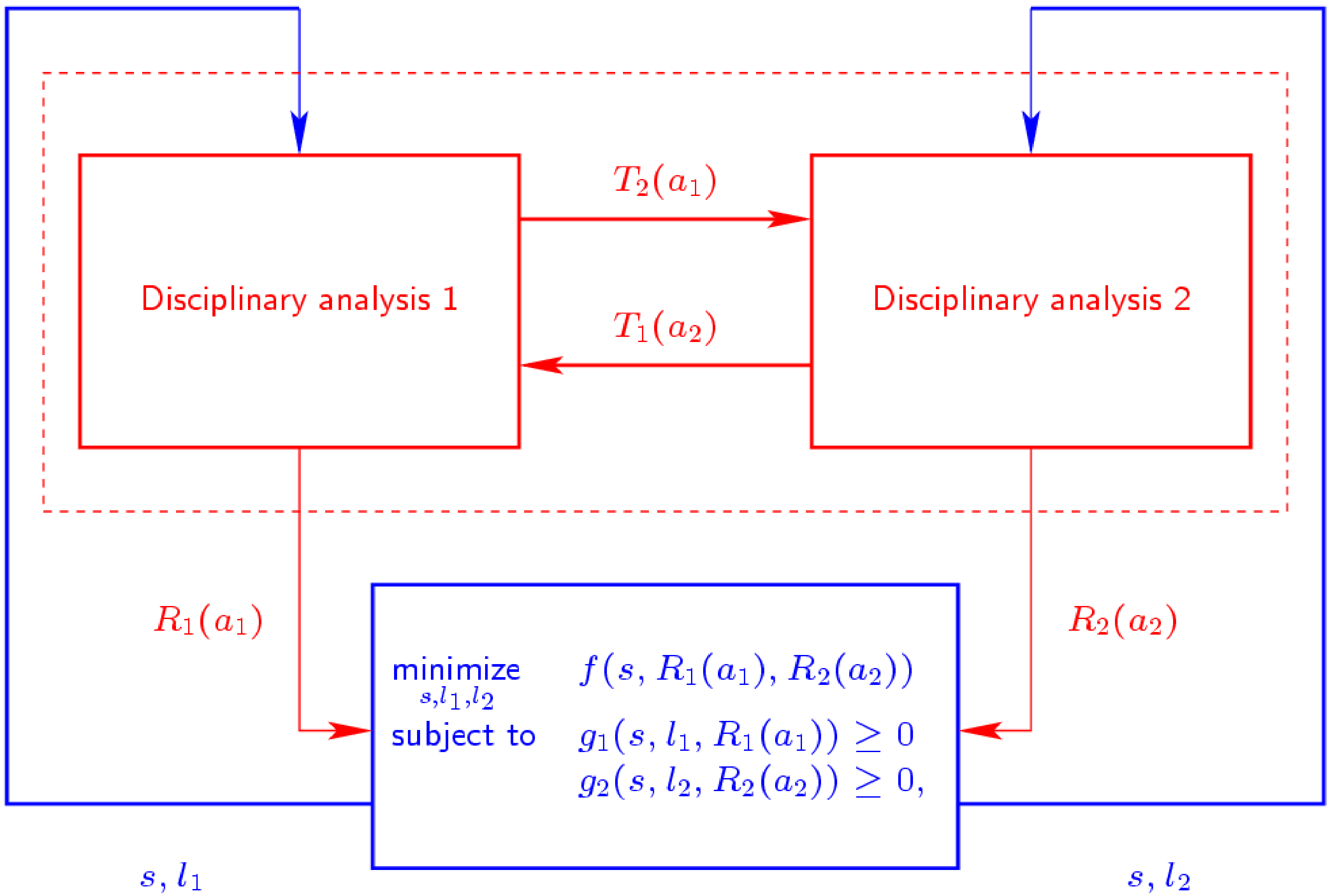
Multidisciplinary analysis: Two discipline example



Multidisciplinary analysis (MDA):

$$\begin{aligned} A_1(a_1, T_1(a_2); s, l_1) &= 0 \\ A_2(a_2, T_2(a_1); s, l_2) &= 0. \end{aligned}$$

Fully integrated optimization (FIO)



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- Requires MDA capability.
- Limited disciplinary autonomy.

Fully integrated optimization (FIO)

The NLP:

$$\begin{array}{ll} \underset{s,l_1,l_2}{\text{minimize}} & f(s, R_1(a_1(s,l_1,l_2)), R_2(a_2(s,l_1,l_2))) \\ \text{subject to} & g_1(s, l_1, R_1(a_1(s,l_1,l_2))) \geq 0 \\ & g_2(s, l_2, R_2(a_2(s,l_1,l_2))) \geq 0, \end{array}$$

where $a_1(s,l_1,l_2)$, $a_2(s,l_1,l_2)$ solve the MDA:

$$\begin{array}{rcl} A_1(a_1, T_1(a_2); s, l_1) & = & 0 \\ A_2(a_2, T_2(a_1); s, l_2) & = & 0. \end{array}$$

Simultaneous analysis and design (SAND)

Relax all coupling in the problem; all variables are independent:

$$
\begin{array}{ll}
\underset{s,l_1,l_2,a_1,a_2}{\text{minimize}} & f(s, R_1(a_1), R_2(a_2)) \\
\text{subject to} & g_1(s, l_1, R_1(a_1)) \geq 0 \\
& g_2(s, l_2, R_2(a_2)) \geq 0 \\
& A_1(a_1, T_1(a_2), s, l_1) = 0 \\
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- Intermediate designs may not be physically sensible.
- Role of disciplinary autonomy not clear.

Distributed analysis optimization (DAO)

Relax the coupling in the MDA, but preserve the disciplinary analyses:

$$
\begin{array}{cl}
\underset{s,l_1,l_2,r_1,r_2,t_1,t_2}{\textbf{min}} & f(s,\, r_1, r_2) \\
\textbf{s.t.} & g_1(s,\, l_1,\, r_1) \geq 0 \\
& g_2(s,\, l_2,\, r_2) \geq 0 \\
& r_1 = R_1(a_1(s, l_1, t_1)) \quad t_1 = T_1(a_2(s, l_2, t_2)) \\
& r_2 = R_2(a_2(s, l_2, t_2)) \quad t_2 = T_2(a_1(s, l_1, t_1)),
\end{array}
$$

where $a_1 = a_1(s, l_1, t_1)$, $a_2 = a_2(s, l_2, t_2)$ satisfy the disciplinary analyses:

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\begin{aligned}
A_1(a_1;\, t_1, s, l_1) &= 0 \\
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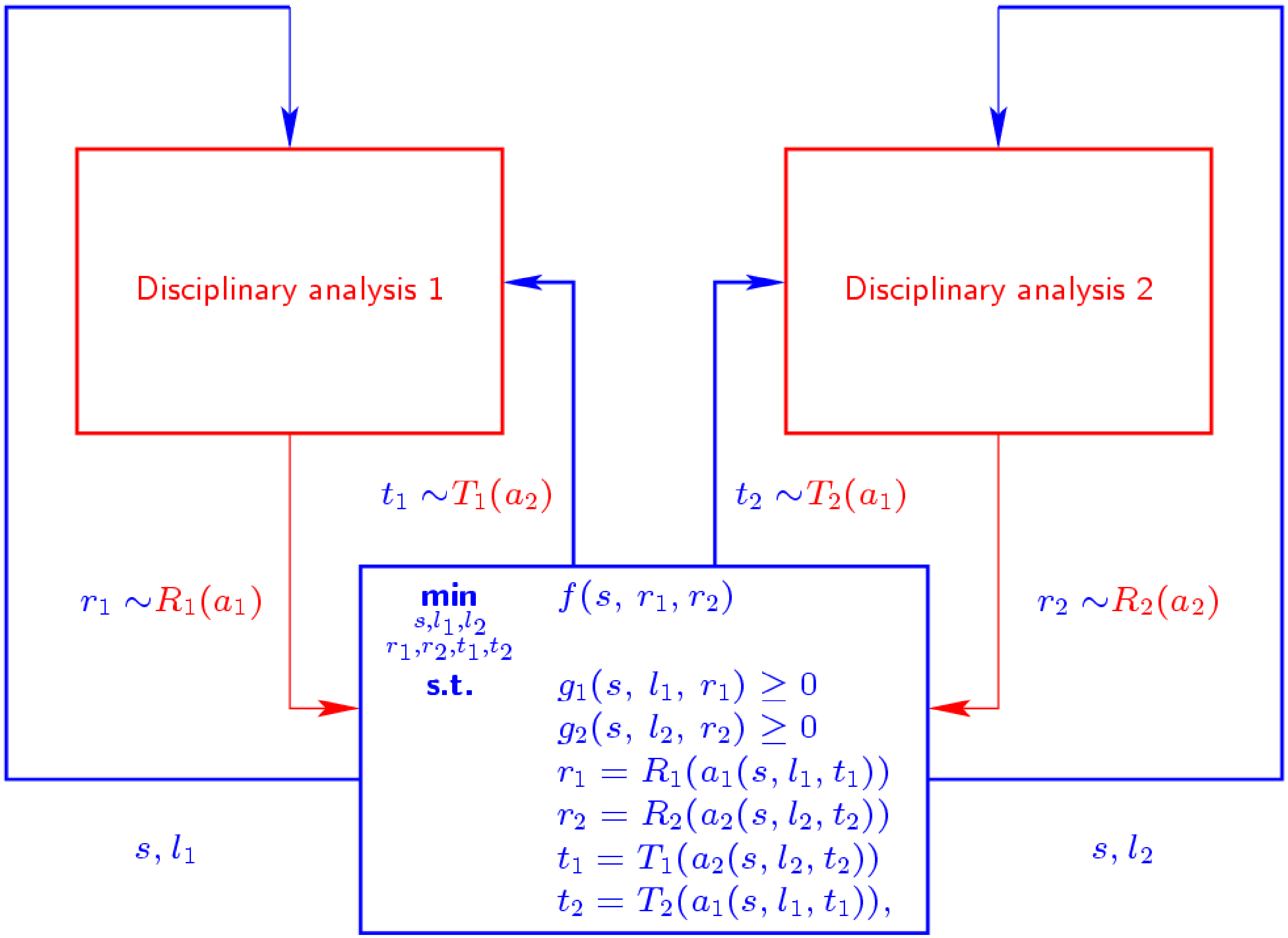

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DAO is similar to the Individual Discipline Feasible (IDF) formulation.

Hierarchy of formulations

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Introduce coupling variables (a_{12}, a_{21}),
$+$ eliminate (a_1, a_2) via disciplinary analyses
$+$ eliminate (l_1, l_2) via disciplinary design constraints $\Rightarrow$???

Elimination of local design variables

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An illustration is an approach sometimes called collaborative optimization.

System-level problem:

$$\begin{aligned} \min_{s,r_1,r_2,t_1,t_2} \quad & f(s,r_1,r_2) \\ \text{s.t.} \quad & c(s,r_1,r_2,t_1,t_2) = 0, \end{aligned}$$

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Disciplinary problems:

$$(1) \qquad \begin{array}{ll} \min\limits_{l_1} & \| \, r_1 - R_1(a_1(s,l_1,t_1)) \, \|^2 + \| \, t_2 - T_2(a_1(s,l_1,t_1)) \, \|^2 \\ \textbf{s.t.} & g_1(s,l_1,a_1(s,l_1,t_1)) \geq 0 \\ \textbf{where} & a_1 = A_1(s,l_1,t_1). \end{array}$$

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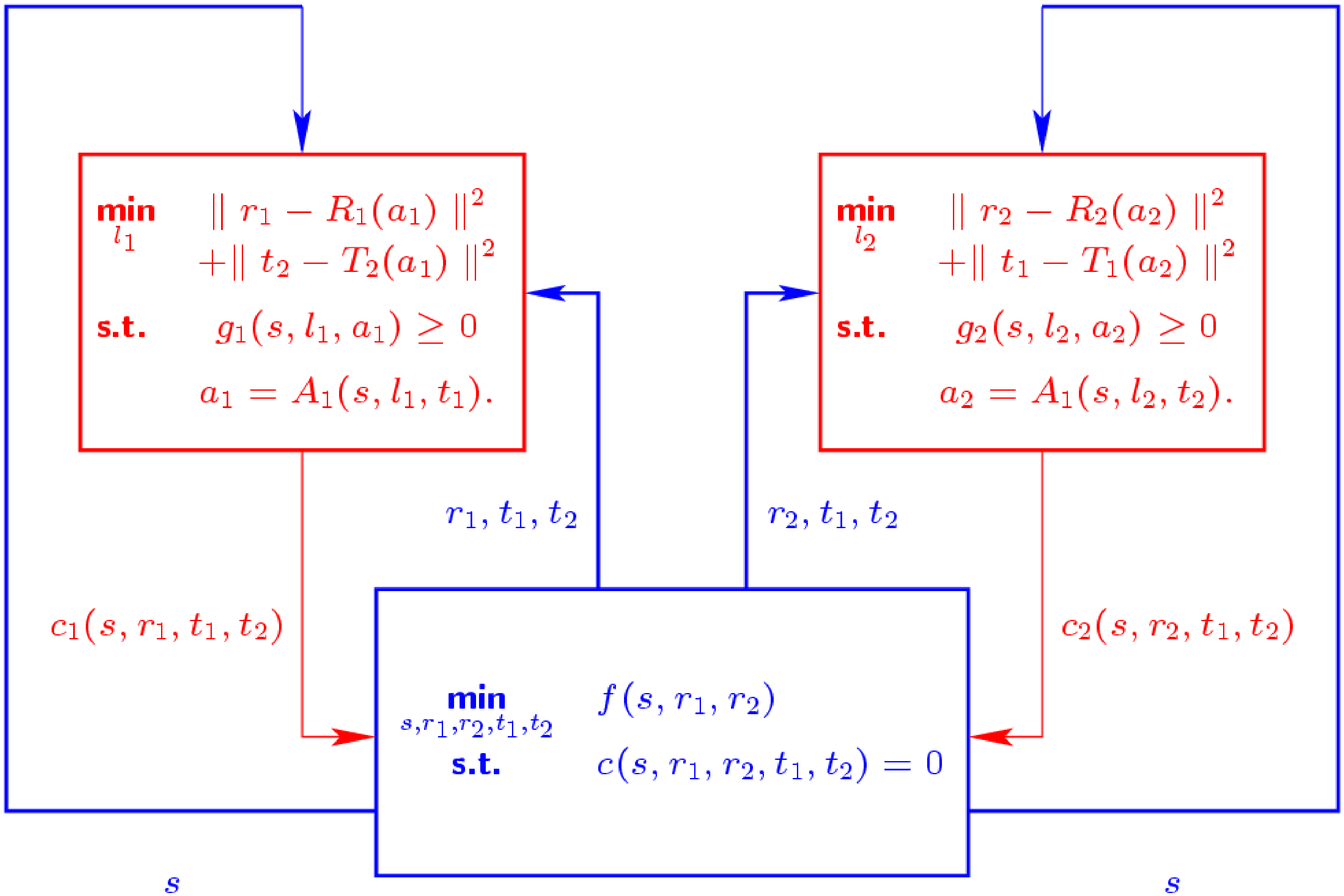
$$(2) \quad \begin{array}{lc} \min\limits_{l_2} & \| \, r_2 - R_2(a_2(s,l_2,t_2)) \, \|^2 + \| \, t_1 - T_1(a_2(s,l_2,t_2)) \, \|^2 \\ \text{s.t.} & g_2(s,l_2,a_2(s,l_2,t_2)) \geq 0 \\ \text{where} & a_2 = A_2(s,l_2,t_2). \end{array}$$

Elimination of local design variables via CO



System-level constraints

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One choice of system-level consistency constraints is:

$$\begin{aligned} c_1(s, r_1, r_2, t_1, t_2) &= \| r_1 - R_1(a_1(s, \bar{l}_1, t_1)) \|^2 + \| t_2 - T_2(a_1(s, \bar{l}_1, t_1)) \|^2 \\ c_2(s, r_1, r_2, t_1, t_2) &= \| r_2 - R_2(a_2(s, \bar{l}_2, t_2)) \|^2 + \| t_1 - T_1(a_2(s, \bar{l}_2, t_2)) \|^2 \end{aligned}$$

$c_i(s, r_1, r_2, t_1, t_2) = 0 \Leftrightarrow$ Discipline i can find local design variables $\bar{l}_i$ such that the analysis output is consistent with the targets r_i and t_i.

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\end{aligned}
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We call such values of s, r_i, t_i *realizable* designs for Discipline i.

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- Disciplinary autonomy exercised in auxiliary disciplinary optimization problems.
- Bilevel program.
- Badly behaved system-level problem.

Analytical difficulties in eliminating local design variables

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We believe this explains computational difficulties reported with bilevel problem formulations.

(NMA and RML; *AIAA Journal (2001)*,
Optimization and Engineering (to appear))

What is possible?

CONJECTURE. In general, an algorithm for MDO can possess at most two of the following three attributes:

- Computational autonomy.
- Computational efficiency.
- Computational robustness.