

# Robust Airfoil Optimization in High Resolution Design Space

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# Outline

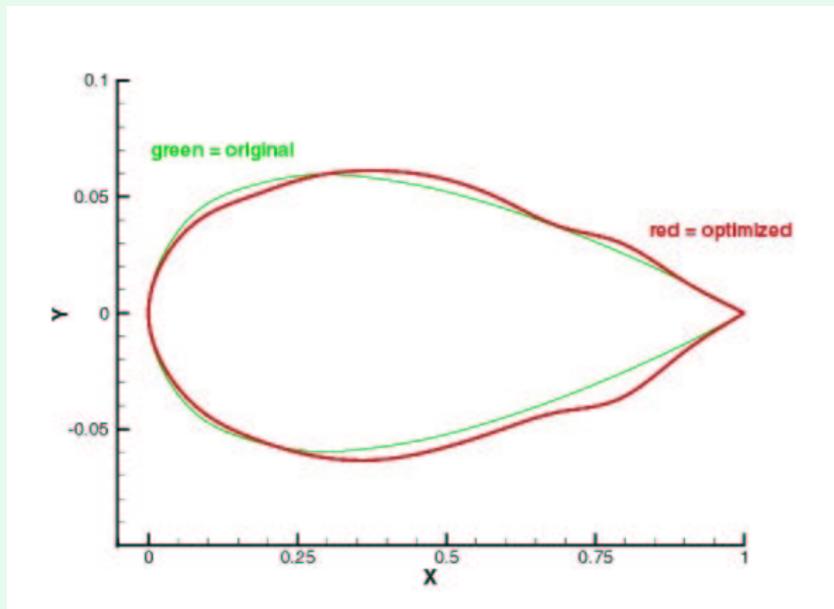


- Airfoil shape optimization
- Robust optimization model for airfoil optimization over a range of flight conditions
- Reduction of mean and variance
- Profile optimization method
  - Adaptive minimax approach for simultaneous and proportional drag reduction
  - Efficient shape modification strategy
- Simulation results in transonic viscous flow
- Conclusions

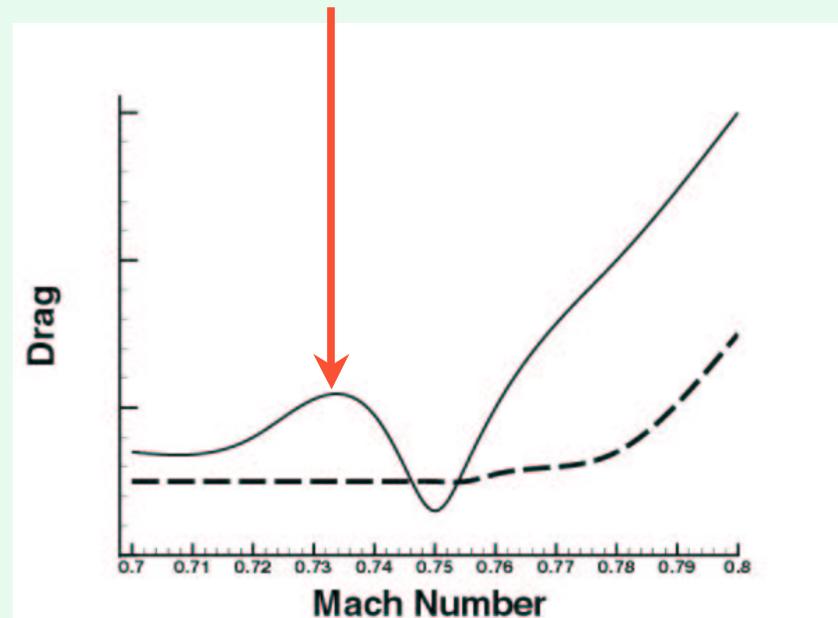
# Two Known Problems Associated With Multipoint Drag Minimization

$$\min_{D, \alpha_1, \dots, \alpha_r} \sum_{i=1}^r w_i c_d(D, M_i, \alpha_i) \text{ st } c_l(D, M_i, \alpha_i) \geq c_l^* \text{ for } 1 \leq i \leq r.$$

Noisy optimal airfoil



Off-design performance degradation



# Past Efforts on Airfoil Shape Optimization

- Smoothing of airfoil shapes during optimization iterations by using smoothed descent directions (Jameson et al)
- Using physical parameters or smooth shape functions (such as Hicks-Henne bump functions) for shape representation
- Multipoint drag minimization or multiobjective optimization (Drela; Nemec, Zingg, and Pulliam)
- Robust optimization methods to avoid off-design performance degradation (Huyse, Lewis, Padula, and Li)

# Robust Optimization Model

Consider the following multi-objective optimization problem:

$$\min(E(c_d), \sigma^2(c_d))$$

(mean, variance)  
with respect to  
Mach number  $M$

subject to

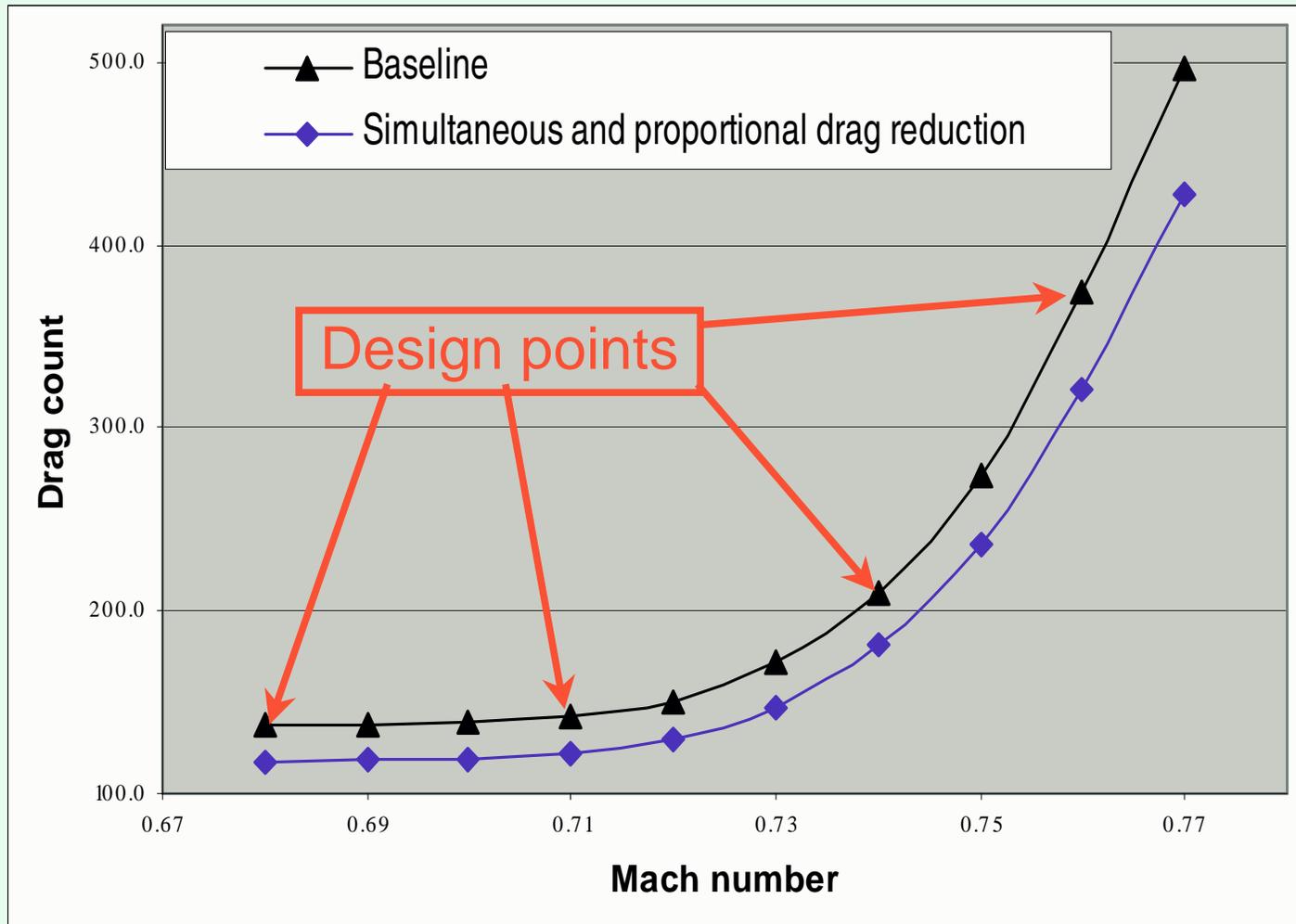
$$c_l(D, \alpha(M), M) = c_l^*(M) \text{ for } M_{\min} \leq M \leq M_{\max}$$

and some geometric constraints (such as thickness constraints).

# Reduction of Mean and Variance

- Use a multi-objective optimization method (such as the weighted sum method) to find a solution.
- However, these methods require fairly accurate approximations of mean and variance of drag.
- For aerodynamic shape optimization in transonic viscous flow, computation of drag is quite expensive. Accurate approximations of mean and variance of drag are computationally prohibitive.

# Our Approach to Reduction of Mean and Variance in Mach Range



# Profile Optimization Method

- 1) **Select design Mach numbers  $M_1, M_2, \dots, M_r$ .**
- 2) **Evaluate the lift and drag, and their gradients.**
- 3) **Find an optimal trust region size for a linear subproblem to achieve simultaneous and proportional drag reduction.**
- 4) **Compute the least norm solution of the linear subproblem.**
- 5) **Generate a new iterate.**

# Choose a Trust Region and Compute the Least Norm Solution

Find the smallest  $\delta$  such that the optimal objective function value of the following linear subproblem is  $(1-\gamma_{\min})$ :

$$\min_{\Delta D, \Delta \alpha_i} \max_{1 \leq i \leq r} \frac{c_d^{\text{predict}}(\Delta D, \Delta \alpha_i, M_i)}{c_d(D, \alpha_i, M_i)} \quad \text{(linearized drag)}$$

subject to

$$c_l^{\text{predict}}(\Delta D, \Delta \alpha_i, M_i) = c_{l,i}^* \quad \text{(linearized lift constraint)}$$

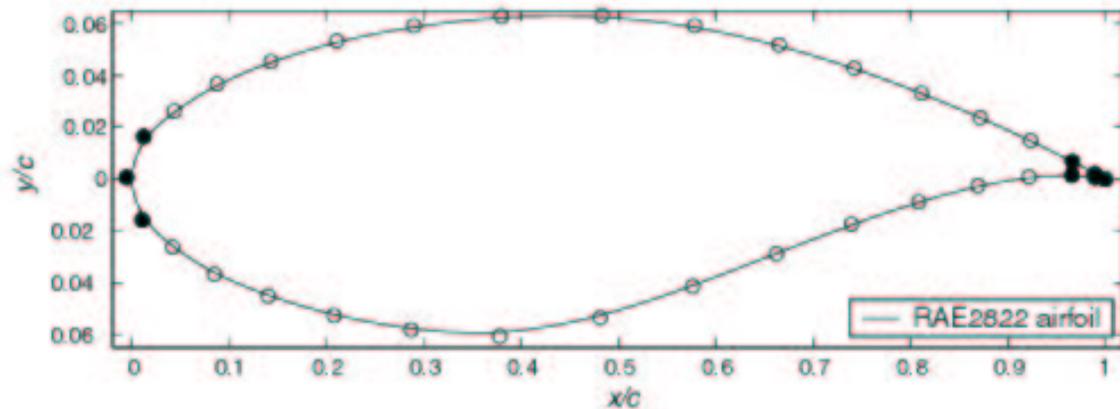
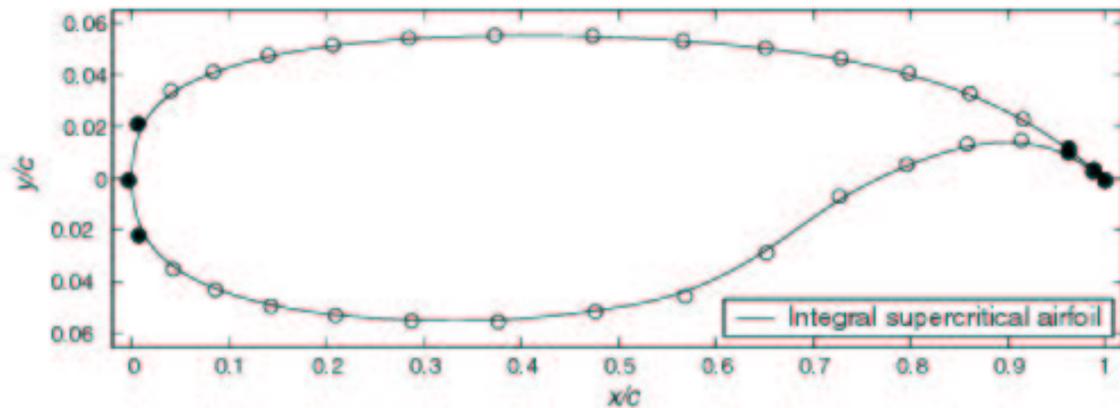
$$-|D_j| \delta \leq \Delta D_j \leq |D_j| \delta \quad \text{(trust region)}$$

and some geometric constraints (such as thickness constraints). Then find the least norm solution of the subproblem.

# Robust Optimization Simulation Results

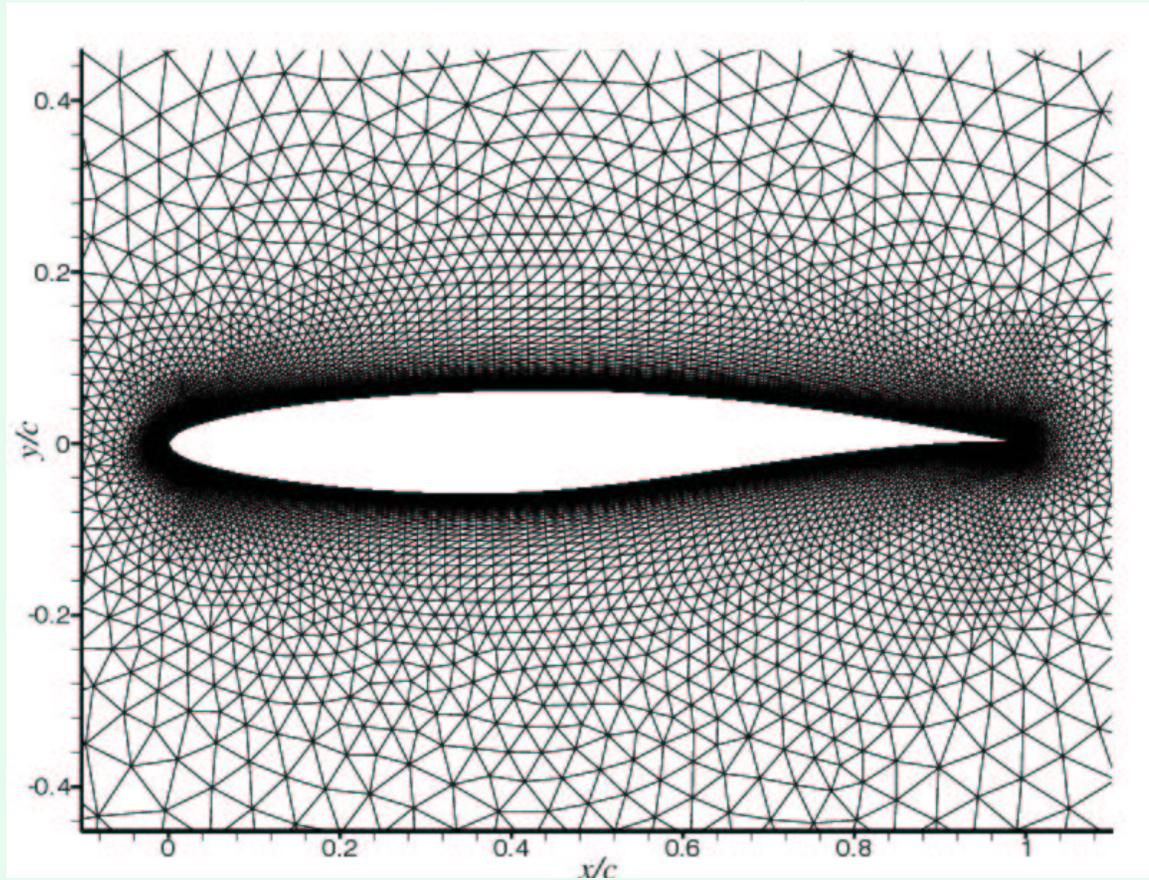
- Whitcomb's supercritical airfoil under the following design conditions:
  - Target lift is 0.7
  - Design Mach numbers:  
**0.68, 0.71, 0.74, 0.77**
- RAE2822 airfoil under Drela's design conditions:
  - Target lift is 0.733
  - Design Mach numbers:  
**0.68, 0.71, 0.74, 0.76**

# Free-Form Airfoil Parameterization Using 35 Cubic B-Spline Control Points



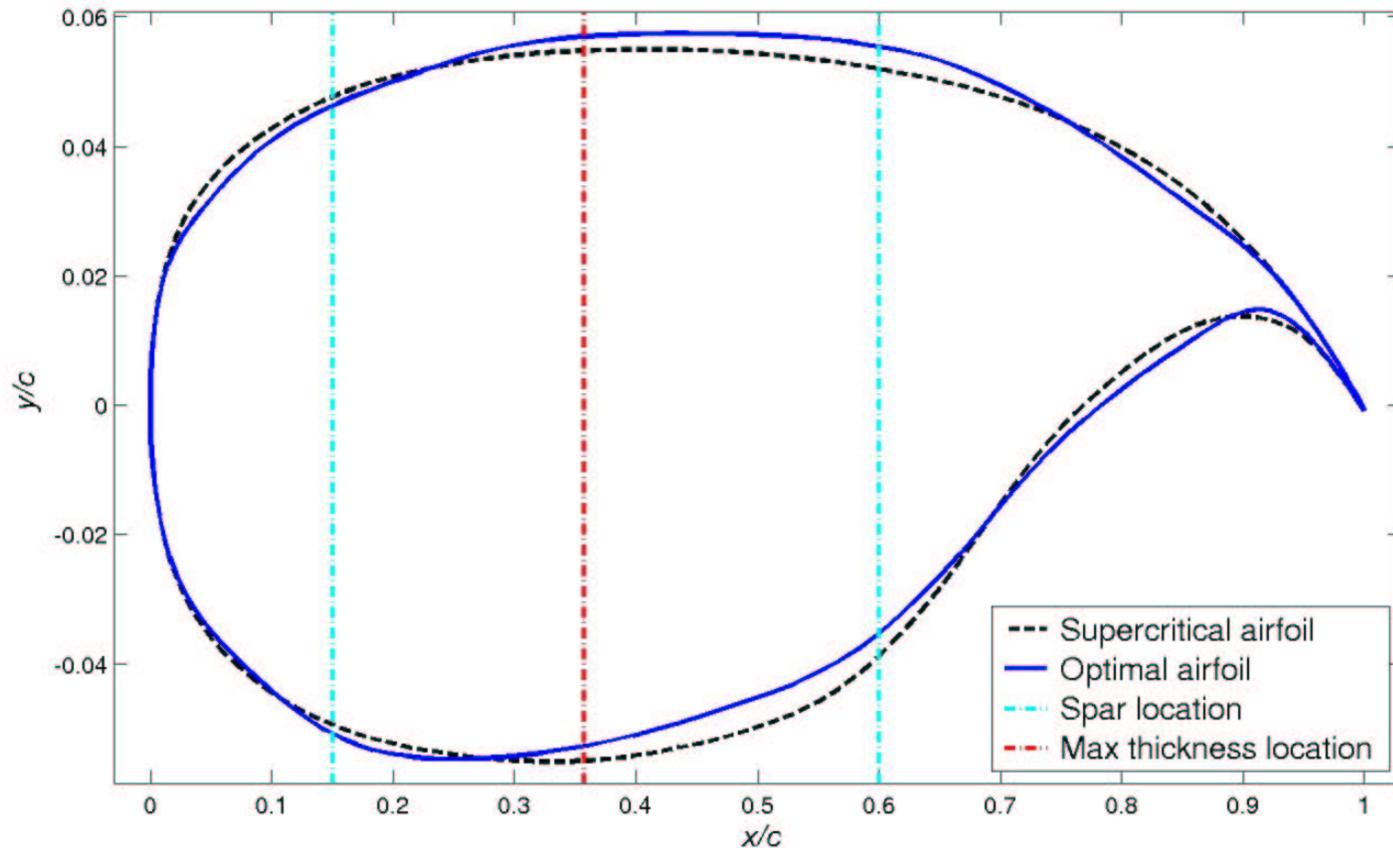
All  $x$ -coordinates are fixed, all  $y$ -coordinates (except the ones marked with black circles) are design variables.

# Unstructured Grid for Solving Navier-Stokes Equations by FUN2D



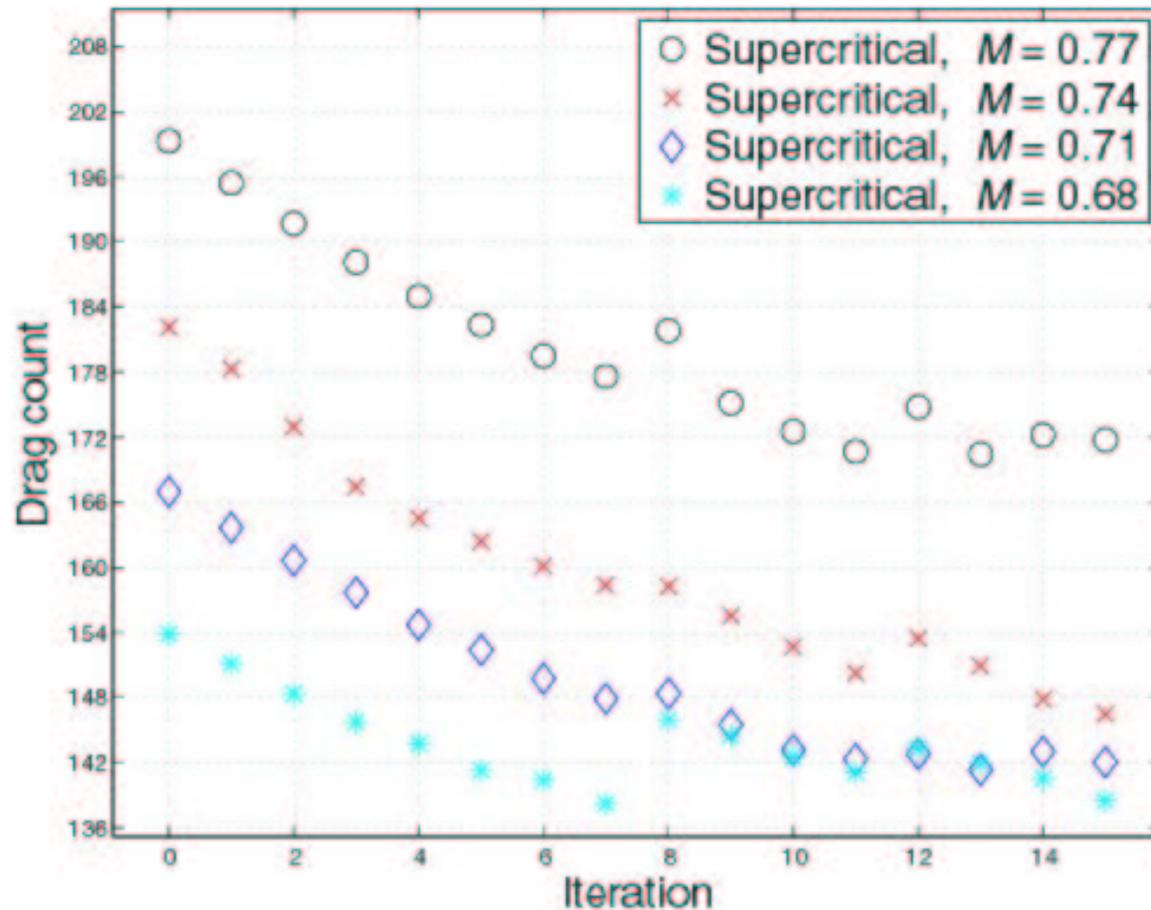
- 18654 grid points, 55631 elements, 37308 faces
- Eight computer nodes are used to run simulation
- Wall-time for generating a new airfoil: 80 minutes

# Optimal Airfoil: Supercritical Airfoil



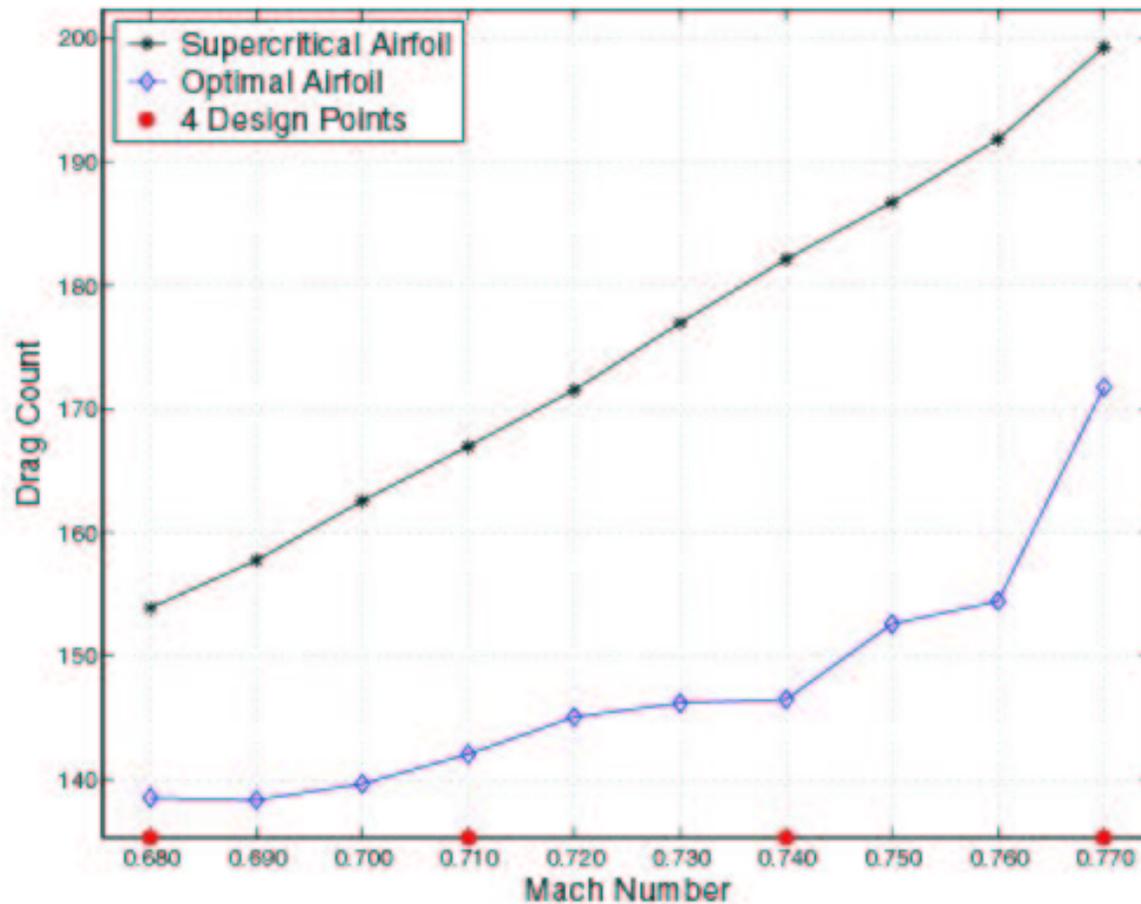
Thickness constraints at two spar locations and the maximum thickness location are used during the optimization.

# Iteration History: Supercritical Airfoil ( $\gamma_{\min}=3\%$ )



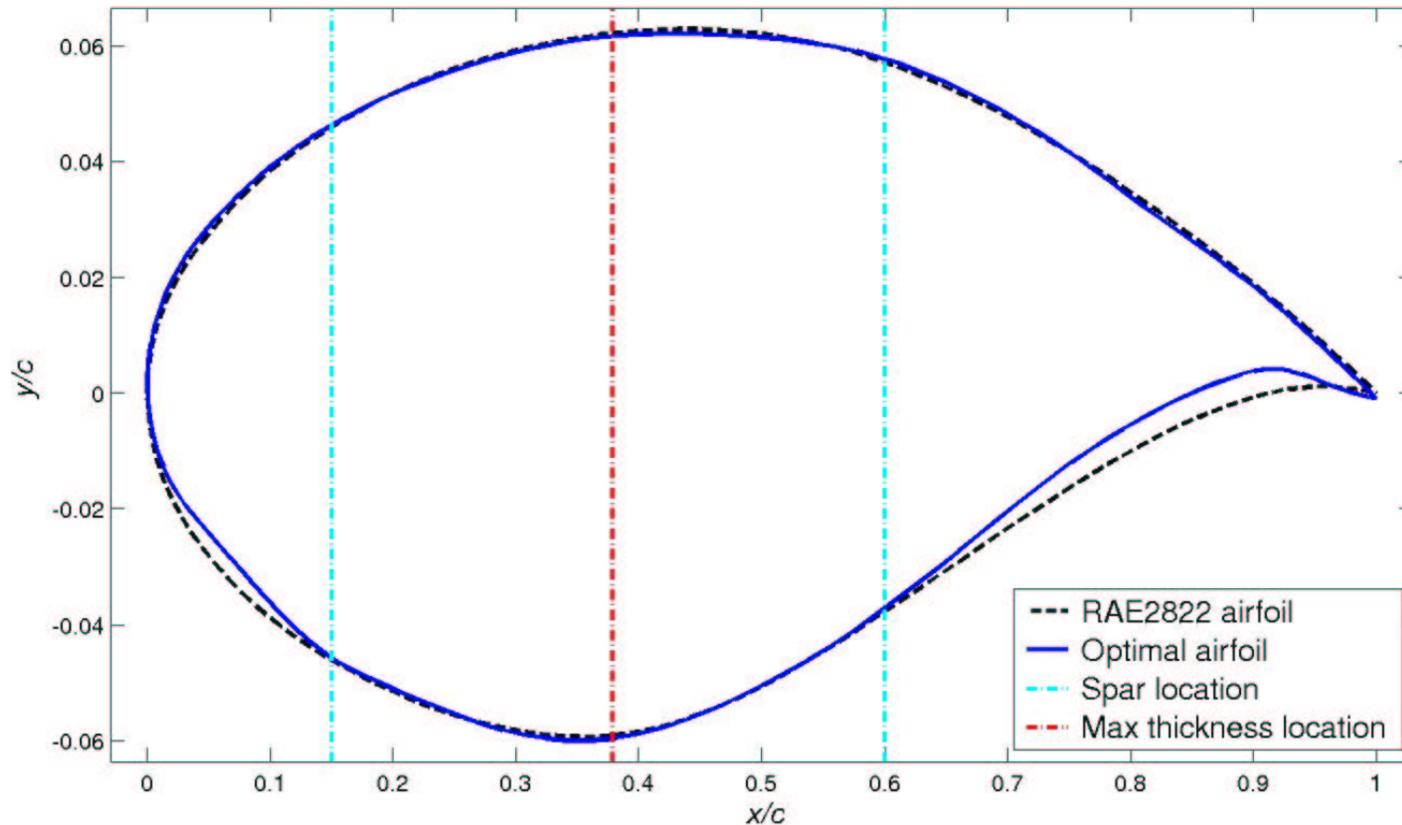
Possible reasons for jumps in drag history: large step length, nonsmooth shape modification, inaccurate linear predictions

# Post Optimization Analysis: Supercritical Airfoil



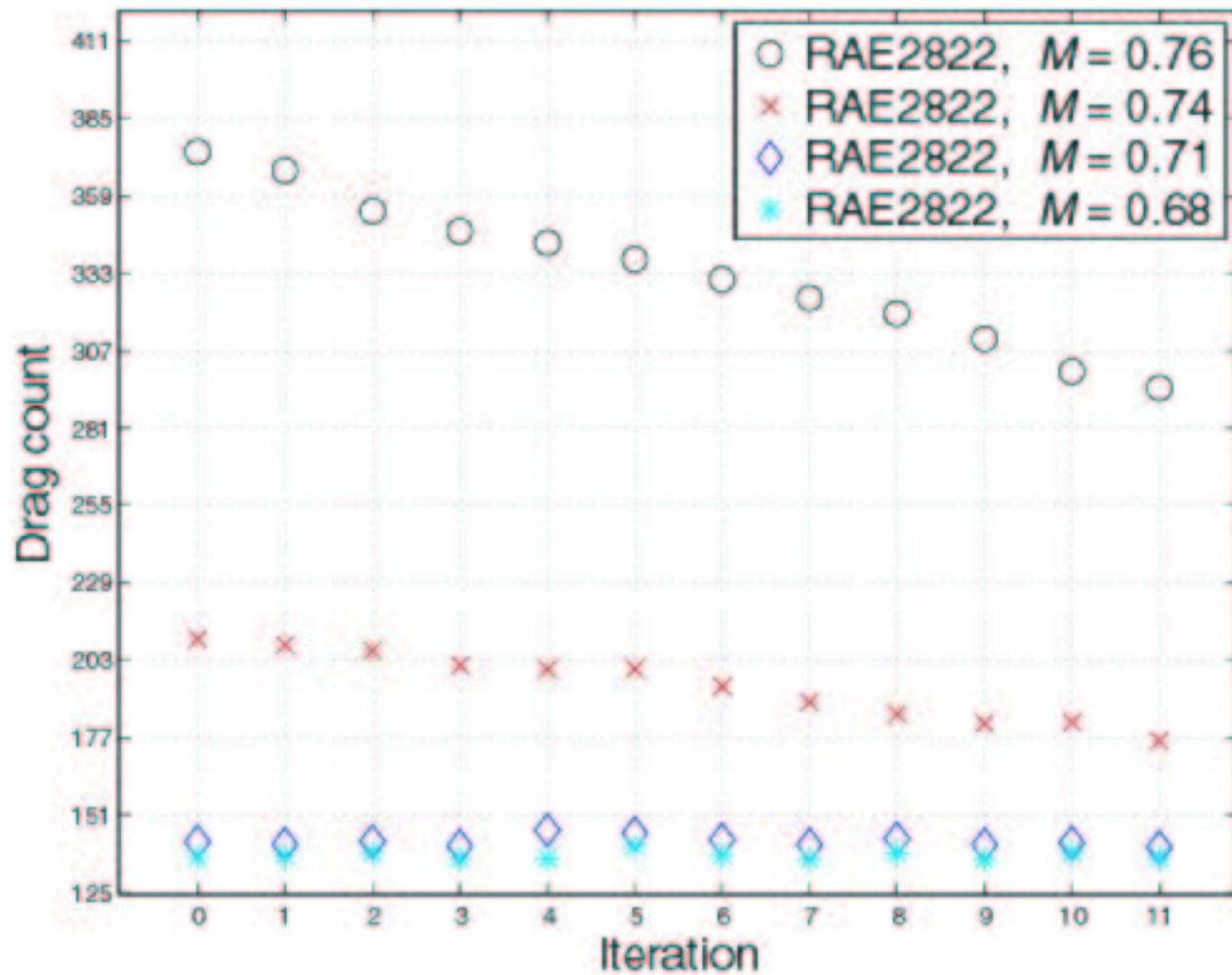
**Drag rise curve with data at 4 design points and 6 off-design points.**  
**Airfoil smoothing may yield a smoother drag rise curve.**

# Optimal Airfoil: RAE2822

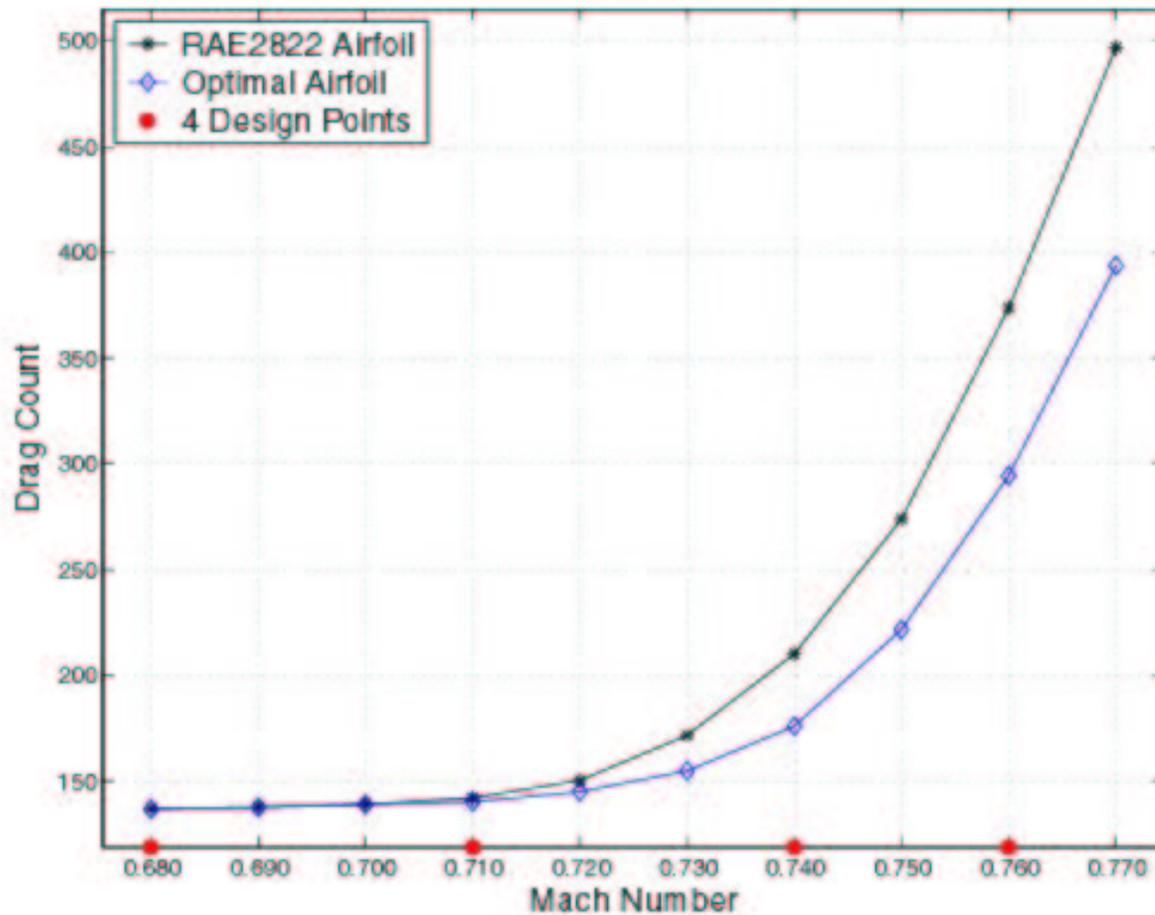


Thickness constraints at two spar locations and the maximum thickness locations are used during the optimization.

# Iteration History: RAE2822 Airfoil ( $\gamma_{\min}=3\%$ )



# Post Optimization Analysis: RAE2822



Drag rise curve with 4 design points and 6 off-design points.

# Concluding Remarks

- By reducing the drag at the design conditions simultaneously and proportionally, the profile optimization method (POM) generates fairly smooth and realistic optimal airfoils without off-design performance degradation, even if free-form parameterization of airfoils is used.
- The computational cost of POM is the same as the multipoint optimization method.

# Future Research Directions

- Incorporate airfoil smoothing in POM
- Develop more flexible drag reduction strategies while enforcing robust optimization policy
- Use more flexible thickness constraints that allow thickness locations to change during the optimization process
- Demonstrate the feasibility of POM for wing design

# Acknowledgments

We would like to thank Eric Nielsen and Mark Chaffin for their assistance with using FUN2D to generate the simulation results. We are also grateful to Richard Campbell and Steven Krist for many stimulating discussions on aerodynamic shape design problems.

Further Info at <http://mdob.larc.nasa.gov>